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Role of the elasticity number in the entry flow of dilute polymer solutions in micro-fabricated contraction geometries

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Abstract

We explore the interplay of fluid inertia and fluid elasticity in planar entry flows by studying the flow of weakly elastic solutions through micro-fabricated planar contraction geometries. The small characteristic lengthscales make it possible to achieve a wide range of Weissenberg numbers ($0.4 < Wi < 42$) and Reynolds numbers ($0.03 < Re < 12$), allowing access to a large region of *Wi*–*Re* space that is typically unattainable in conventional macroscale entry flow experiments. Experiments are carried out using a series of dilute solutions ($0.78 < c/c^* < 1.09$) of a high molecular weight polyethylene oxide, in which the solvent viscosity is varied in order to achieve a range of elasticity numbers, $2.8 < El = Wi/Re < 68$. Fluorescent streak imaging and micro-particle image velocimetry (μ-PIV) are used to characterize the kinematics, which are classified into a number of flow regimes including Newtonian-like flow at low *Wi*, steady viscoelastic flow, unsteady diverging flow and vortex growth regimes. Progressive changes in the centreline velocity profile are used to identify each of the flow regimes and to map the resulting stability boundaries in *Wi*–*Re* space. The same flow transitions can also be detected through measurements of the enhanced pressure drop across the contraction/expansion which arise from fluid viscoelasticity. The results of this work have significant design implications for lab-on-a-chip devices, which commonly contain complex geometric features and transport complex fluids, such as those containing DNA or proteins. The results also illustrate the potential for using micro-fabricated devices as rheometric tools for measuring the extensional properties of weakly elastic fluids.

Keywords: Non-newtonian; Microfluidics; Particle image velocimetry; Contraction flow; Polyethylene oxide

1. Introduction

In macroscale devices (i.e. geometries in which the characteristic lengthscale is on the order of millimeters), it is essentially impossible to generate large deformation rates and correspondingly high Weissenberg numbers (*Wi*) in low viscosity elastic fluids, whilst also maintaining small Reynolds numbers (*Re*). As a result, it is difficult to induce an elastic response in which the effects of viscoelasticity are not dampened (or completely quashed) by the competing effects of fluid inertia. Microfluidic devices offer a solution by allowing high deformation rates and concomitantly low Reynolds numbers; a result that is directly attributable to the small lengthscale of the device.

Several recent studies have shown that the reduced lengthscales associated with microfluidic devices (on the order of tens to hundreds of microns) can enhance the magnitude of viscoelastic effects in dilute polymer solutions. This has been demonstrated in micro-fabricated converging or planar contraction geometries by Groisman and Quake [1] and in the recent work of Rodd et al. [2]. The same phenomena were also observed in the much earlier work of James and Saringer [3] at similar lengthscales and using similar aqueous solutions of flexible polymers. The importance of the device lengthscale and its effect on fluid elasticity is reflected in the definition of the elasticity number, $El = \lambda\eta/(\rho l^2)$, which is dependent only on fluid properties (relaxation time, solution viscosity, and fluid density) and the characteristic lengthscale of the device, *l*.

In addition to the unique flow conditions attainable by scaling down the geometry, microfluidic devices also offer the advantage of allowing access to a greater range of *Wi* and *Re*. This has been shown in our previous work [2], in which elasticity numbers spanning almost two orders of magnitude could be achieved. Accessibility to wide regions of *Wi*–*Re* space provides an avenue for generating suitable experimental data to test the performance

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of constitutive models over a wide range of flow conditions (with and without inertia). Furthermore, the ability of achieving high *Wi* at low *Re* offers the possibility of devloping microfluidic rheometers suitable for probing the rheological properties of weakly elastic fluids such as inks or dilute polymer solutions that appear Newtonian under the conditions that can be attained in conventional rheometers [2,4].

Very few experiments have been conducted specifically to test the effect of the elasticity number on complex viscoelastic flows, which is primarily attributed to the limited range of parameter space accessible through macro-scale experiments. With regards to planar contraction flows, the most thorough investigations of the effect of the elasticity number have been achieved through numerical simulations (see Table 1). We have previously provided a broader survey of experimental works in [2]; however in Table 1 we focus on planar flows which specifically investigate at least one of the following: (i) planar versus axisymmetric geometries, (ii) the effect of the elasticity number and (iii) the role of the viscoelastic Mach number, $Ma = \sqrt{Re\,Wi}$. In addition, many of the references in Table 1 also provide numerical predictions of the centreline velocity and/or extensional viscosity predictions.

To our knowledge, Rodd et al. [2] is the only experimental study which provides at least preliminary insight into the effect of *El* on the non-linear dynamics of planar entry flows. However, the range of values of the elasticity number in [2] was achieved by varying the polymer concentration, which is expected to lead to additional non-linear rheological effects associated with variable chain–chain interactions.

In the present work, we investigate the flow of four dilute polyethylene oxide solutions ($0.78 < c/c^* < 1.09$) through a micro-fabricated abrupt contraction–expansion geometry (contraction ratio, CR = 16), in which the smallest lengthscale of the device is 26 μm in the throat of the contraction. A range of elasticity numbers ($2.8 < El < 68$) are achieved by varying the solvent viscosity whilst maintaining a constant polymer concentration in solution ($c = 0.075$ wt.%). Experiments are performed over a range of flow conditions corresponding to $0.03 < Re < 12$ and $0.4 < Wi < 42$. Fluorescent streak imaging, micro-particle image velocimetry and pressure drop measurements are used to characterize the upstream flow kinematics associated with steady and time-dependent three-dimensional flow for both the elastic solutions and a Newtonian fluid, and to evaluate the extra pressure drop due to the elasticity of the solutions. Lastly, we assess the importance of the viscoelastic Mach number [5,6], and its role in determining the onset of diverging flow in this set of low viscosity elastic solutions.

1.1. *Flow phenomena in viscoelastic entry flows*

1.1.1. *Planar versus axisymmetric geometries*

It has been shown, both experimentally and numerically, that the kinematics associated with entry flows in planar and axisymmetric geometries are inherently quite different. For shear-thinning elastic fluids in planar contraction geometries, elastic corner vortices grow with increasing *Wi*; however the extent of vortex growth within a planar geometry [7–10] is less than in the equivalent axisymmetric geometry [11]. Table 1 identifies cases in which numerical simulations have been able to reproduce either qualitatively or quantitatively the results of specific experimental studies.

For Boger fluids however, vortex growth has not been observed in macro-scale planar contractions. Experimentally, Nigen and Walters [12] found (through both pressure drop measurements and streakline images) that for low to moderate flowrates, there is no discernable difference between the upstream flow dynamics in a Boger fluid and a Newtonian fluid in a 16 to 1 planar contraction. A number of 2D numerical simulations of flow through planar contractions for an Oldyroyd-B fluid [13,14] or an upper-convected Maxwell fluid [15–20], all lead to the same conclusion; the size of the corner vortex decreases with increasing Weissenberg number. However, higher values of *Wi* have been found to lead to the formation of unstable lip vortices. This has been observed both experimentally [12] and numerically [14,17].

The only case in which elastic corner vortex growth in Boger fluids has been observed in planar contractions has been in the recent experimental results of Rodd et al. [2]. In their work, micro-fabricated planar contractions were used in conjunction with a set of low viscosity Boger fluids in order to induce vortex growth, however this was only observed at moderate Reynolds numbers ($Re > 11$).

The reduced magnitude of elastic vortex growth that is observed experimentally in planar geometries, compared with their axisymmetric counterpart, is commonly attributed to the reduced strain rate in the geometry and/or the reduced total Hencky strain that is experienced by a polymer molecule as it flows through the contraction ($\epsilon_{\text{axi}} = 2\ln \text{CR}$, compared with $\epsilon_{\text{planar}} = \ln \text{CR}$) [21]. However, even for high contraction ratios, non-linearities in the dynamic response have been found to be virtually absent in planar geometries [22]. Changing the contraction ratio by adjusting the upstream channel width results in an increase in the total Hencky strain however this extra contribution only occurs in the upstream tail of the strain rate profile, i.e. regions in which the strain rate is typically small and less than the critical value, $\dot{\epsilon}_{\text{crit}} = 1/\lambda$, required for polymer extension. As a result, the Hencky strain that is accumulated in high strain rate regions that actually lead to chain extension remains unchanged [22]. It is therefore the non-homogeneity of the strain rate profile observed in planar contraction flows that is considered responsible for the lack of non-linearity in the stress-response. This observation was made by Genieser et al. based on birefringence measurements in Boger fluids and 1D predictions using the Geisekus model, and the upper-convected and linear Maxwell models [22]. Their arguments however, do not explain off-centreline dynamics, such as the sustained vortex growth observed in shear-thinning viscoelastic fluids.

Quinzani et al. also made point-wise flow-induced birefringence measurements in a shear-thinning viscoelastic fluid flowing through a 4:1 planar contraction [23,24]. Although they quantify in great detail the fluid velocity, shear stress and first normal stress difference as a function of spatial position, their measurements were only carried out in a planar geometry, precluding any direct comparison of the corresponding extensional stresses induced in planar and axisymmetric geometries for the