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Turbulent channel flow of dilute polymeric solutions: Drag reduction scaling and an eddy viscosity model

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Abstract

Direct numerical simulation of viscoelastic turbulent channel flows up to the maximum drag reduction (MDR) limit has been performed. The simulation results in turn have been used to develop relationships between the flow and fluid rheological parameters, i.e. maximum chain extensibility, Reynolds number, Re_τ, and Weissenberg number, We_τ and percent drag reduction (%DR) as well as the slope increment of the mean velocity profile. Moreover, based on the trends observed in the mean velocity profile and the overall momentum balance three different regimes of drag reduction (DR), namely, low drag reduction (LDR; $0 \leq$ %DR ≤ 20), high drag reduction (HDR; $20 \leq$ %DR ≤ 52) and MDR ($52 \leq$ %DR ≤ 74) have been identified and mathematical expressions for the eddy viscosity in these regimes are presented. It is found that both in LDR and HDR regimes the eddy viscosity varies with the distance from the channel wall. However, in the MDR regime the ratio of the eddy viscosity to the Newtonian one tends to a very small value around 0.1 within the channel. Based on these expressions a procedure that relies on the DNS predictions of the budgets of momentum and viscoelastic shear stress is developed for evaluating the mean velocity profile.

Keywords: Turbulent drag reduction; Slope increment; Reynolds stress; Viscoelastic; FENE-P; Direct numerical simulation (DNS); Eddy viscosity model

1. Introduction

Turbulent flows of non-Newtonian liquids are encountered in several engineering applications such as turbulent drag reduction (DR), fire fighting, multiphase reactor design and agricultural spraying. Typically the non-Newtonian behavior results from the modification of fluid rheology by the addition of small amounts of additives such as polymers, fibers or amphiphilic solutes (surfactants) to Newtonian solvents. For instance, the addition of small amounts of soluble high molecular weight polymers to inertia-dominated, wall bounded flows is known to reduce the turbulent drag. It has been observed experimentally that polymer concentration of O(1 0 0) ppm is sufficient to reduce drag up to 70%. This has stimulated tremendous research effort in the past 50 years. Comprehensive reviews of early literature on this area are given in Hoyt [1], Lumley [2,3] and Virk [4]. Phenomenological models that capture the salient experimentally observed features of polymer induced turbulent drag reduction have been proposed since the 1960s. For instance, Meyer [5] formulated one of the earliest phenomenological models of DR in which the principal effects of drag-reduced additives are an upward shift of the logarithmic part of the velocity profile and thickening of the viscous sublayer. Virk [6] later proposed an improved model suggesting that the mechanism responsible for drag reduction must take place somewhere between the viscous sublayer and the logarithmic zone which he named the “elastic sublayer”. Virk [6] suggested that as drag reduction increases the width of this layer should also increase and, at maximum drag reduction (MDR; %DR ~ 70–80%), the elastic sublayer could occupy the entire logarithmic region.

In the past decade, the development of accurate and efficient spectral methods for viscoelastic turbulent flow simulation and their implementation on high performance computers have made it possible to investigate turbulent drag reduction in dilute polymer solutions using first principle constitutive models derived from polymer kinetic theory. Such direct numerical simulations (DNSs) allows one to probe the influence of fluid rheology, especially the extensional viscosity and relaxation time, on the viscoelastic stress, turbulent fluctuations, coherent structures and DR [7–10]. However, as in the case of Newtonian flows,

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direct numerical simulations of viscoelastic turbulent flows have certain limitations. First of all, the number of primary variables increases by a factor of 2 in the latter. Moreover, as DR increases, the near wall streaks become progressively stabilized and elongated. This necessitates the use of simulation boxes with very large (up to 60*h*, where *h* is the channel half height) streamwise length for high DR values [10]. Furthermore, the time step size limitation is much more stringent due to the hyperbolic nature of the constitutive model and since the diffusive formulations traditionally used in the literature [11] require the solution of six additional Helmholtz equations per time step to update the viscoelastic stress. Consequently, for a given Reynolds number, the CPU-time and memory requirements for the DNS of viscoelastic flows are at least an order of magnitude larger as compared to the Newtonian case. Hence, to date DNS of turbulent DR has been mainly limited to relatively small Reynolds numbers and simple geometries such as fully developed channel flows, and flows past a flat plate of homogeneous polymer solutions [7–13]. However, in several applications (e.g. boundary layer flows past ships) the Reynolds number can be of the order of 10^9. DNSs are impractical even for Newtonian flows at such large Reynolds numbers since the number of degrees of freedom in DNS $\propto Re^{9/4}$. In such situations, for Newtonian flows, one resorts to modeling approaches such as the development of Reynolds Averaged Navier-Stokes (RANS) equations. [14,15] or large eddy simulations (LES) [16–19]. DNS results at relatively low *Re* can provide modeling guidelines and help validate RANS-based approaches. This has sparked recent research initiatives [20–22] aimed at the development of RANS-like approaches for viscoelastic turbulent flows. One of the prerequisites for the development of faithful and robust RANS models for polymer drag reduced flows is the accurate modeling of the statistical averages of the products of the fluctuations in the velocity (deformation rate) and viscoelastic stress gradients (viscoelastic stress) that arise from the Reynolds-Averaged constitutive equation. There are methodological and computational challenges associated with developing closure expressions for the statistical averages in the Reynolds-Averaged constitutive equation. For instance, recent DNS studies have shown that the flow characteristics undergo qualitative changes as DR is increased. For instance, in the low drag reduction regime (LDR; $\%\mathrm{DR} \leq 30$), the mathematical behavior of the various terms in the Reynolds stress budget is, in general, qualitatively similar to that in the Newtonian flow [23] and the direct contribution of the polymeric stress to the overall momentum balance is relatively small. Consequently, RANS modeling has enjoyed considerable success in this regime [20]. However, as DR is increased, the streamwise vortices are stabilized significantly and the flow structure is qualitatively modified leading to the establishment of a buffer region that remains quiescent for a fairly long period of time punctuated only by occasional (intermittent) turbulence producing events [24,25]. Under such circumstances, the use of modified Newtonian RANS models may not be appropriate. For instance, in the modified Newtonian (LDR) regime, the Taylor microscale, $\lambda = l\,Re_l^{-1/2}$ where l denotes the large eddy scale, used to express the order of magnitude estimates of the gradients in statistical averages is well-defined. In fact the very first DNS in the LDR regime has shown little variation in the spectrum at small scales [7]. This scenario could change qualitatively near MDR where a high degree of intermittency exists.

Despite the aforementioned, unresolved issues with RANS-based modeling, it has been used since the 1970s as a tool for providing engineering estimates of the effect of DR on the mean flow profiles and/or kinetic energy. In the absence of detailed information on the spectral characteristics of drag-reduced flows (except for the recent work on isotropic turbulence by Vaithianathan and Collins [26]), single-point turbulence closures are used. In addition to the prediction of the friction factor, estimation of heat/mass transfer coefficients in polymer drag reduced turbulent flows [25], is also of technological relevance. Although there have been research efforts in this direction [27–30] in the 1970s, DNS studies of scalar transport have been only recently performed [25].

For Newtonian flows, one-equation models, especially modification of the standard k–ε model, have been developed for engineering applications. The standard k–ε model contains five parameters while its modifications could involve additional ones: see Table 4.1 in [31]. While some of these parameters are the same for all the models (due to the constraint that the model predictions should reproduce the experimental and/or DNS results for isotropic/homogeneous turbulence), the improvements introduce additional semi-empirical correlations as well as modifications in the expression for the eddy diffusivity ($\propto k^2/\varepsilon$). In addition, wall functions are necessary to apply these models to flows with solid boundaries. Introduction of viscoelasticity introduces new parameters and cross-correlations (between velocity and viscoelastic stress) [32] whose robust determination is a daunting task. Hassid and Poreh [28] proposed a model based on the turbulent kinetic energy equation and an eddy diffusivity, which is capable of reproducing velocity profiles in drag-reduced flows in the LDR regime though it requires the distribution of turbulent length scales. To overcome this shortcoming, Hassid and Poreh [29] later proposed an alternative model based on a low-Reynolds number version of the k–ε model. Specifically, they modified the two equations for k and ε to account for the increase in turbulent kinetic energy in drag-reduced flows and proposed a new eddy-viscosity damping function, which involves a single adjustable parameter whose value must be determined for each polymer–solvent solution from friction factor data. However, the predictions of the existing viscoelastic k–ε models in isotropic/homogeneous shear flows have not been tested against DNS data and/or experimental results. As the DNS literature in this area [26] becomes available, such model validation could be done and the variability in parameter selection could be potentially minimized.

The phenomenological models proposed in the 1970s for viscoelastic turbulent flows are based on the modification of the empirical parameters in the Newtonian closures. Poreh and Hassid [33] have given a review of the earlier models for drag-reduced flows and compared their predictions with that obtained based on an energy dissipation closure that describes the observed effects of drag reduction on the mean velocity profile and turbulent kinetic energy. However, these closures were unable to describe drag reduction with generality and required