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Transient squeeze flow of viscoplastic materials

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Abstract

The transient, axisymmetric squeezing of viscoplastic materials under creeping flow conditions is examined. The flow of the material even outside the disks is followed. Both cases of the disks moving with constant velocity or under constant force are studied. This time-dependent simulation of squeeze flow is performed for such materials in order to determine very accurately the evolution of the force or the velocity, respectively, and the distinct differences between these two experiments, the highly deforming shape and position of all the interfaces, the effect of possible slip on the disk surface, especially when the slip coefficient is not constant, and the effect of gravity. All these are impossible under the quasi-steady state condition used up to now. The exponential constitutive model, suggested by Papanastasiou, is employed. The governing equations are solved numerically by coupling the mixed finite element method with a quasi-elliptic mesh generation scheme in order to follow the large deformations of the free surface of the fluid. As the Bingham number increases, large departures from the corresponding Newtonian solution are found. When the disks are moving with constant velocity, unyielded material arises only around the two centers of the disks verifying previous works in which quasi-steady state conditions were assumed. The size of the unyielded region increases with the Bingham number, but decreases as time passes and the two disks approach each other. Their size also decreases as the slip velocity or the slip length along the disk wall increase. The force that must be applied on the disks in order to maintain their constant velocity increases significantly with the Bingham number and time and provides a first method to calculate the yield stress. On the other hand, when a constant force is applied on the disks, they slow down until they finally stop, because all the material between them becomes unyielded. The final location of the disk and the time when it stops provide another, probably easier, method to deduce the yield stress of the fluid.

Keywords: Transient squeeze flow; Viscoplastic fluids; Free surface flows

1. Introduction

Squeeze flow is widely used as a typical experiment for the rheological characterization of non-Newtonian fluids [1]. It is also encountered in various engineering processes like compression molding, which is used for the manufacturing of various industrial parts [2]. The rheological experiment can be performed in two different ways, either by measuring the force required to push the disks at a constant velocity [3–5] or by measuring the velocity of the disks towards each other when a constant force is applied [6–8]. Our efforts here are aimed at developing an accurate and efficient numerical method in order to simulate both versions of such a rheological experiment for the special case of a viscoplastic material.

The deformation and flow of viscoplastic materials are very important, since many multicomponent fluids such as suspensions, pastes, paints, foodstuffs, foams and slurries, which are very frequently encountered in industrial processes, are viscoplastic [9]. These materials exhibit small or no deformation at all (solid-like behavior) when the applied stress is below a particular value, which is called yield stress. Above the yield stress, they flow with viscosity that depends on the local rate of strain. Whether yield stress really exists is a subject that has drawn some attention [10], however it is widely accepted that this concept, when it is used properly, can provide an accurate description of physical phenomena and industrial processes.

The first constitutive law that was proposed for describing the flow of such materials is the Bingham model [11]. Its application is extended to more than one dimensions through the Von Mises criterion for dividing the domain occupied by the material in regions where it behaves either as rigid solid or as liquid. When two such regions coexist in a domain this criterion introduces a discontinuity in the constitutive law at the yield surface, which

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is the location where these two regions meet. Moreover, when the flow is multi-dimensional or time-dependent, this discontinuity usually leads to considerable computational problems, because in such cases it is very difficult to predict correctly the location and the shape of the yield surface. It is characteristic that although this model is used extensively for simple and steady flows, very few researchers have used it in more complex flows. Such an exception is the work by Beris et al. [12], who presented a numerical simulation for the creeping flow around a moving sphere. Although at first sight this problem does not seem to be so complicated, it required a very careful analysis in order to reveal that unyielded material arises around the two stagnation points of flow and at some finite distance from the sphere. Alternatively, Frigaard and co-workers have advanced the augmented Lagrangian method as a viable alternative to deal with such problems [13]. To avoid difficulties like these, various modifications of the Bingham constitutive equation have been introduced. Two such modifications are the biviscosity model, proposed by O'Donovan and Tanner [14] and the exponential model, proposed by Papanastasiou [15]. In the present analysis we decided to use the Papanastasiou model which has the following form:

$$\underline{\underline{\tau}}^* = -\left[\eta_0 + \frac{\tau_y(1-\mathrm{e}^{-m\dot{\gamma}^*})}{\dot{\gamma}^*}\right]\underline{\underline{\dot{\gamma}}}^* \qquad (1)$$

where the superscript (*) denotes a dimensional quantity, $\underline{\underline{\tau}}^*$ is the extra stress tensor, η_0 is the plastic viscosity, $\underline{\underline{\dot{\gamma}}}^*$ is the rate of strain tensor defined as $\underline{\underline{\dot{\gamma}}}^* = \underline{\nabla v}^* + (\underline{\nabla v}^*)^{\mathrm{T}}$, $\dot{\gamma}^*$ is its second invariant, $\dot{\gamma}^* = [(1/2)\underline{\underline{\dot{\gamma}}}^* : \underline{\underline{\dot{\gamma}}}^*]^{1/2}$, and m is the stress growth exponent.

Smyrnaios and Tsamopoulos [16] have shown that for relatively large values (depending on the flow at hand) of the exponent coefficient, m, this model closely approximates the discontinuous Bingham behavior. On the other hand, Burgos et al. [17] have suggested that extremely large values of this coefficient should be avoided for reasons that have to do with numerical stability and the stiffness of the resulting discrete system. The main advantage of this constitutive equation is the fact that it is continuous and it holds uniformly in yielded and unyielded regions. Thus the determination of the shape and the location of the yield surface can be performed a posteriori instead of simultaneously with the flow, as required by the discontinuous Bingham model [12]. We explain the method we have followed for its determination in Section 3.4. Moreover, this model has a continuous dependence of the stress on $\underline{\underline{\dot{\gamma}}}^*$, which has been argued to be more meaningful physically [18]. On the other hand, the yield surface, for example, is expected not to coincide completely with that determined following the discontinuous Bingham model.

Many researchers have employed the aforementioned models in order to simulate the viscoplastic behavior in complex or even in time-dependent flows, but we will mention here only some characteristic examples. Jay et al. [19] have used the biviscosity model to study the flow through a sudden axisymmetric expansion and provided some very interesting comparisons with experimental observations. The Papanastasiou model has been used in transient simulations by Tsamopoulos et al. [20] to study the thinning of a viscoplastic fluid film on a rotating disk in the process of spin coating and by Dimakopoulos and Tsamopoulos [21] to study the displacement of a viscoplastic material by air in straight and suddenly constricted tubes.

The behavior of viscoplastic materials in squeeze flow has attracted the attention of several researchers in the past. In early theoretical studies, the most usual approach was the lubrication approximation. This approach is still used today to evaluate experimental results, although it was noticed almost from the beginning that it led to profound kinematic inconsistencies in the calculated velocity fields [16,22]. O'Donovan and Tanner [14] were the first who recognized the need to solve the squeeze film problem without using the lubrication approximation. They solved numerically the constant velocity problem employing the biviscosity model for describing the viscoplastic behavior of the material and they finally concluded that unyielded material arises only adjacent to the center of the plates. Quite recently Smyrnaios and Tsamopoulos [16] provided a qualitative analysis and accurate numerical simulations for this problem assuming quasi-steady state conditions. They employed both the original Bingham constitutive equation and the exponential one to definitively clarify all the misleading speculations about the position and the existence of yield surfaces in this flow field. They showed that unyielded material could only exist around the two stagnation points of flow extending in this way the work by O'Donovan and Tanner [14]. Matsoukas and Mitsoulis [23] also solved numerically the squeeze flow of viscoplastic materials, assuming quasi-steady state, for both planar and axisymmetric flow, confirming the earlier results by Smyrnaios and Tsamopoulos [16]. They have also provided a more accurate correlation for the force that must be applied on the disks' surface in order to maintain their constant velocity. The only transient simulations of non-Newtonian fluids undergoing squeeze flow that we are aware of are those by Mavridis et al. [24], albeit for a power-law fluid.

In the present work, we solve the transient squeeze flow of a viscoplastic material for both cases where the disks are moving with constant velocity and under constant force. Clearly, this distinction is impossible under the quasi-steady state conditions. The transient simulation enables the determination of the transient force or velocity, respectively, the shape of the liquid/air interface, and the location of the yield surface, at every time instant. Moreover we are able to study the effect on the process of possible slip of the fluid on the disk surface and also the effect of gravity.

The remainder of the paper is organized as follows. In Section 2, we present the governing equations and the boundary conditions for this problem. The numerical algorithm, used in our simulations, is described in Section 3. In Section 4, we present the results of the extensive parametric analysis that we performed for a viscoplastic fluid that is being squeezed either with constant disk velocity or under constant force. Finally, conclusions of the present study are drawn in Section 5.