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# Effect of Lorentz forces on forced-convection nanofluid flow over a stretched surface

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ABSTRACT

Magnetic nanofluid hydrothermal analysis over a plate is studied that includes consideration of thermal radiation. The Runge–Kutta (RK4) method is utilized to get solution of ODEs which are obtained from similarity solution. In considering the impacts of Brownian motion, we applied Koo–Kleinstreuer–Li correlation to simulate the properties of CuO–water. The influence is discussed of important parameters such as the temperature index, magnetic, radiation, and velocity ratio parameters and volume fraction of nanoparticle on hydrothermal behavior. Results illustrate that the coefficient of skin friction enhances with enhancing magnetic parameter while reduces with enhancing velocity ratio parameter. Also the Nusselt number was found to directly depend on the velocity ratio and temperature index parameters but has an inverse dependence on the magnetic and radiation parameters.



## Introduction

Heat transfer across boundary layers of stretching plates finds uses in polymers, extrusion, cooling, and rotating of fibers. The cooling procedures should be controlled because the feature of the last product relies on heat transfer rate. Crane (1970) investigated the flow over a plate which was stretched. He presented a solution for velocity profile. The nonlinear stretching plate has been presented by Afzal and Varshney (1980). Then Afzal (1993) studied the heat transfer of previous problem. He considered different boundary condition for temperature profile. Influence of suction on the stretching plate in existence of heat generation has been studied by Elbashbeshy and Bazid (2004). Metallurgical procedure and reactor cooling can be considered as examples of magnetohydrodynamic flow uses. Newtonian Magnetohydrodynamic flow over a plate has been investigated by Chakrabarti and Gupta (1979). Two dimensional magnetohydrodynamic flows between two plates was presented by Borkakoti and Bharali (1983).

Nanofluid technology can be considered as one of passive method for enhancing heat transfer. Magnetic nanofluid flow in an L-shape cavity was examined by Sheikholeslami, Ganji, Gorji-Bandpy, and Soleimani (2014e). Their results indicated that heat transfer enhances with rise of buoyancy forces. Nanofluid flow over permeable plate has been presented by Rashidi, Abelman, and FreidooniMehr (2013). They showed that rate of heat transfer can be improved by applying magnetic field. Buongiorno model has been applied for heatline analysis of free convection by Sheikholeslami, Gorji-Bandpy, and Soleimani (2013d). They indicated that buoyancy ratio number has different influences on Nusselt number. Flow of magnetic nanofluid material through two plates which were rotated has been investigated by Sheikholeslami, Hatami, and Ganji (2014d). They showed that heat transfer rate enhances with adding more nanoparticle. Garoosi, Bagheri, and Talebi (2013) examined the numerical method for free convection of nanofluid. They found the optimum value for volume fraction in their geometry. Application of nanofluid has been reviewed by Wen, Lin, Vafaei, and Zhang (2009). The uses of nanofluid have been indicated by several authors (Hatami & Ganji, 2014a; Hatami & Ganji, 2014b; Hayat, Shehzad, & Alsaedi, 2012a; Hayat, Shehzad, Qasim, & Obaidat, 2012b; Mustafa, Hina, Hayat, & Alsaedi, 2012; Shehzad, Alsaedi, & Hayat, 2012; Sheikholeslami & Ganji, 2013, 2014a–c; Sheikholeslami & Gorji-Bandpy, 2014; Sheikholeslami, Gorji-Bandpy, & Domairry, 2013c; Sheikholeslami, Gorji-Bandpay, & Ganji, 2012; Sheikholeslami, Gorji-Bandpy, & Ganji, 2013a, 2014b; Parker, 2009; Sheikholeslami et al., 2014e; Sheikholeslami, Gorji-Bandpy, Ganji, Rana, & Soleimani, 2014f).

Hu, Wang, and Song (2009) presented experimental results for synthesis of magnetite nanoparticles. Nanofluid mixed convection in an enclosure was considered by Garoosi, Garoosi, and

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**Nomenclature**

| | |
|---|---|
| $A_i\,(i=1\ldots5)$ | *constants parameters* |
| $a$ | stretching plate index |
| $b$ | velocity index |
| $C_f$ | coefficient of skin friction |
| $k$ | thermal conductivity |
| $M$ | magnetic parameter |
| $Nu$ | Nusselt number |
| $Rd$ | radiation parameter |
| $T_\infty$ | ambient temperature |
| $T$ | temperature |
| $(u, v)$ | horizontal and vertical velocities |
| $(x, y)$ | horizontal and vertical directions |

*Greek symbols*

| | |
|---|---|
| $\theta$ | dimensionless temperature |
| $\eta$ | similarity parameter |
| $\phi$ | nanoparticle volume fraction |
| $\lambda$ | velocity ratio parameter |
| $\mu$ | dynamic viscosity |
| $\sigma_e$ | Stefan–Boltzmann constant |
| $\sigma$ | electrical conductivity |

*Subscripts*

| | |
|---|---|
| nf | nanofluid |
| $\infty$ | Infinity condition |
| $w$ | surface condition |

Hooman (2014). They showed that as diameter of nanoparticle enhances, *Nu* reduces. Magnetohydrodynamic nanofluid has been investigated by different authors (Hayat, Ahmed, Sajid, & Asghar, 2007; Hayat, Hussain, & Khan, 2006; Hatami, Sheikholeslami, & Ganji, 2014; Sheikholeslami & Ganji, 2014d; Sheikholeslami, Gorji-Bandpy, & Ganji, 2013b, 2014c; Sheikholeslami, Gorji-Bandpay, & Ganji, 2014a; Sheikholeslami, Ashorynejad, & Rana, 2016a; Sheikholeslami, Vajravelu, & Rashidi, 2016b).

From our current study, we report on the magnetic field impact in CuO–water nanofluid over a stretching plate. Thermal radiation influence is considered. The Koo–Kleinstreuer–Li (KKL) model is utilized to estimate CuO–water properties (Li, 2008; Koo & Kleinstreuer, 2004; Koo & Kleinstreuer, 2005). The parameter dependence for the problem was studied and is discussed.

## Mathematic description

Fig. 1 depicts the schematic of the present study. Stretching and free stream velocities are $U_w(x)$ and $U_\infty(x)$, respectively and equal to $ax$ and $bx$. Temperature of the plate is $T_w(x) = T_\infty + cx^n f$. Also, magnetic field ($B_0$) is applied. Furthermore, the effects from thermal radiation are considered in this problem.

Single phase model is utilized for simulate nanofluid. Table 1 illustrates the properties of water and CuO. The governing partial different equations (PDEs) and boundary conditions are as follows:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \tag{1}$$

**Table 1**
Thermo physical properties of water and nanoparticles at room temperature (Li, 2008).

| | $\rho$ (kg/m$^3$) | $C_p$ (J/kg K) | $k$ (W/m k) | $d_p$ (nm) |
|---|---|---|---|---|
| Pure water | 997.1 | 4179 | 0.613 | – |
| CuO | 6500 | 540 | 18 | 29 |

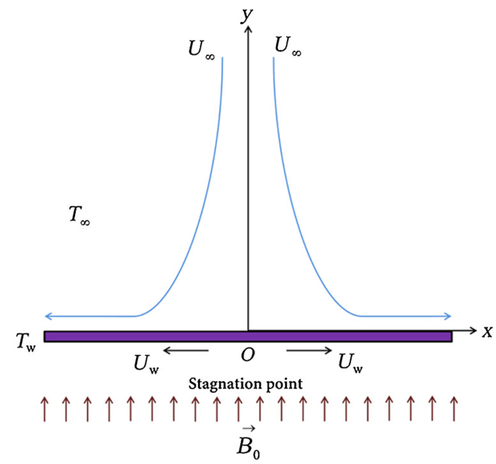


**Fig. 1.** Schematic of forced-convection nanofluid flow over a stretching surface.

$$\rho_{nf}\left(-U_\infty\frac{dU_\infty}{dx} + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial x}u\right) = \mu_{nf}\frac{\partial^2 u}{\partial y^2} + \sigma_{nf}{B_0}^2(-u+U_\infty), \tag{2}$$

$$(\rho C_p)_{nf}\left(\frac{\partial T}{\partial x}u + \frac{\partial T}{\partial y}v\right) = -\frac{\partial q_r}{\partial y} + k_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$\begin{aligned} &av = 0, \quad T = T_w(x), \quad u = U_w(x) \quad @y = 0 \\ &T = T_\infty, \; u = U_\infty(x) \qquad\qquad @y \to \infty \end{aligned} \tag{4}$$

where $v$ and $u$ are the vertical and horizontal velocities, respectively. The radiation heat flux $q_r$ obeys the Rosseland approximation $q_r = -\left(4\sigma_e/3\beta_R\right)\left(\partial T^4/\partial y\right)$ where $\beta_R$, $\sigma_e$ are the coefficient of mean absorption and the Stefan–Boltzmann constant, respectively. Temperature differences are small enough, so according to Taylor series $T^4$ can be written as $T^4 \cong 4T_c^3 T - 3T_c^4$

The effective the electrical conductivity, density, and heat capacitance of the CuO–water are obtained as (Sheikholeslami et al., 2012):

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3(-1+\sigma\sigma)\phi}{-(-1+\sigma\sigma)\phi + (+2+\sigma\sigma)}, \quad \sigma\sigma = \frac{\sigma_s}{\sigma_f} \tag{5}$$

$$(\rho C_p)_{nf} = +(\rho C_p)_s\phi + (\rho C_p)_f(1-\phi), \tag{6}$$

$$\rho_{nf} = \rho_s\phi + \rho_f(1-\phi), \tag{7}$$

$k_{nf}$ and $\mu_{nf}$ of CuO–water are simulated using KKL model (Li, 2008; Koo & Kleinstreuer, 2004, 2005)

$$k_{nf} = k_{static} + k_{Brownian}, \tag{8}$$

$$\frac{k_{static}}{k_f} = 1 + \frac{3\left(\left(k_p/k_f\right)-1\right)\phi}{\left(\left(k_p/k_f\right)+2\right) - \left(\left(k_p/k_f\right)-1\right)\phi}, \tag{9}$$

$$R_f + \frac{d_p}{k_p} = \frac{d_p}{k_{p,eff}}, \quad R_f = 4\times10^{-8}\ \mathrm{km^2/W} \tag{10}$$

$$k_{Brownian} = 5\times10^4\phi\rho_f c_{p,f}\sqrt{\frac{\kappa_b T}{\rho_p d_p}}g'(T,\phi,d_p). \tag{11}$$

$$\begin{aligned} g'(T,\phi,d_p) = &\left(a_1 + a_2\ln(d_p) + a_3\ln(\phi) + a_4\ln(\phi)\ln(d_p) + a_5\ln(d_p)^2\right)\ln(T) \\ &+ \left(a_6 + a_7\ln(d_p) + a_8\ln(\phi) + a_9\ln(\phi)\ln(d_p) + a_{10}\ln(d_p)^2\right), \end{aligned} \tag{12}$$

$$\mu_{eff} = \frac{\mu_f}{(1-\phi)^{2.5}} + \frac{k_{Brownian}}{k_f}\frac{\mu_f}{Pr_f} \tag{13}$$

The coefficients of CuO–water are presented in Table 2.