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Experimental study on the 3D vibrated packing densification of binary sphere mixtures

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ABSTRACT

The packing densification of binary spherical mixtures under 3D mechanical vibration was studied experimentally. The influences of vibration frequency (ω), volume fraction of large spheres (X_L), sphere size ratio (r, diameter ratio of small to large spheres), and container size (D) on the random binary packing density (ρ) were systematically analyzed. For any given set of conditions, there exist optimal ω and X_L to realize the densest random binary packing; too large or small ω and X_L is not helpful for densification. The influences of both r and D on ρ are monotonic; either reducing r or increasing D leads to a high value of ρ. With all other parameters held constant, the densest random packing occurs when X_L is dominant, which is in good agreement with the Furnas relation. Moreover, the highest random binary packing density obtained in our work agrees well with corresponding numerical and analytical results in the literature.

Introduction

Particle packing is an important subject in both scientific research and many industrial applications (German, 1989; Bideau & Hansen, 1993). Among various particle shapes, the 3D packing of spheres and 2D packing of disks have attracted the most attention. This is because they can be effectively used as the simplest models to understand the static structure of more complicated systems, analogous to simple liquids, amorphous and crystalline solids, which correspond to three critical packing states: random loose packing (packing density, $\rho \approx 0.60$), random close packing ($\rho \approx 0.64$), and ordered packing ($\rho \approx 0.74$) (Rintoul & Torquato, 1996). Therefore, in the past decades, a large amount of experimental and numerical work has been conducted on the packing densification of monodisperse spheres to identify the densest random (Scott, 1962; Scott, Charlesworth, & Mak, 1964; Ayer & Soppet, 1965; Scott & Kilgour, 1969; Finney, 1970, 1977; Woodcock, 1976; Clarke & Jónsson, 1993; Knight, Fandrich, Lau, Jaeger, & Nagel, 1995; Torquato, 2000; Stachurski, 2003; An, Yang, Dong, Zou, & Yu, 2005; An, Li, Yang, Zou, & Yu, 2009; Wu, An, & Huang, 2014) and ordered (Owe Berg, Mcdonald, & Trainor, 1970; Rocke, 1971; Pouliquen, Nicolas, & Weidman, 1997; Van Blaaderen, Ruel, & Wiltzius, 1997; Blair, Mueggenburg, Marshall, Jaeger, & Nagel, 2001; Nahmad-Molinari & Ruiz-Suárez, 2002; Spannuth, Mueggenburg, Jaeger, & Nagel, 2004; Yu, An, Zou, Yang, & Kendall, 2006; Li, An, Yang, Zou, & Yu, 2011; An, Yang, Dong, & Yu, 2011; An & Huang, 2013; Panaitescu & Kudrolli, 2014) packing structures and their transitions. The ordered packing of face-centered cubic (FCC) or hexagonal close-packed (HCP) structures with a packing density of about 0.7405 is now regarded as the densest packing of monodisperse hard spheres. This was first conjectured by Kepler 400 years ago and has recently been proved mathematically (Szpiro, 2003; Hales, 2005).

In addition to the large amount of literature dealing with the packing densification of monodisperse spheres, a considerable number of studies on the random packing densification of binary sphere mixtures have been carried out both numerically (Visscher & Bolsterli, 1972; Clarke & Wiley, 1987; Frost, Schön, & Salamon, 1993; Rouault & Assouline, 1998; Danisch, Jin, & Makse, 2010; Desmond & Weeks, 2009, 2014; Hopkins, Stillinger, & Torquato, 2013) and analytically (Yerazunis, Cornell, & Wintner, 1965; Yu & Standish, 1987, 1991; Lochmann, Oger, & Stoyan, 2006; Brouwers, 2013). In contrast, despite some physical experiments having been conducted (McGeary, 1961; Epstein & Young, 1962; Ridgway & Tarbuck, 1968; Jeschar, Potke, Petersen, & Polthier, 1975; Zou, Feng, & Yu, 2001), few studies have focused on the densest random binary packings of spheres subjected to 3D mechanical vibrations. In particular, comprehensive investigations of the influences of various parameters, such as the vibration frequency ω, sphere size ratio

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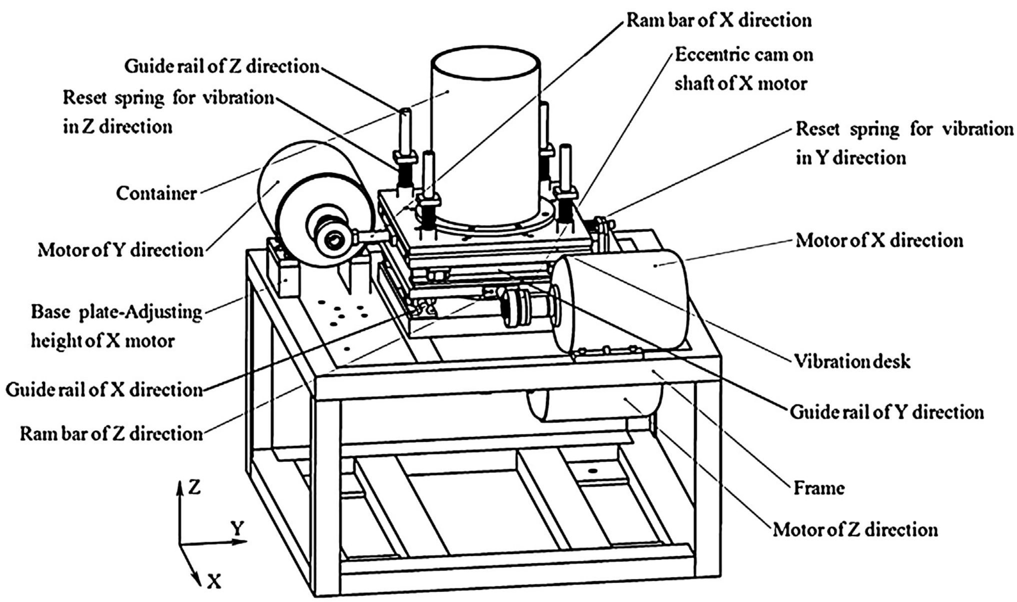

Fig. 1. Schematic representation of the vibration setup.

r (defined as $r = d_s/d_L$, where d_s and d_L represent the diameter of the small and large spheres, respectively), composition X_L (volume fraction of large spheres with respect to the total sphere volume), and container size D (wall effects), on the packing density of binary random sphere packings are lacking. Therefore, systematic experiments on the densification of binary random sphere mixtures under 3D vibrations were carried out in the present study with the objective being: (a) to identify the role of various factors in vibrated sphere packing densification; (b) to attain the highest packing densities of binary random packing mixtures; and (c) to obtain the optimal processing parameters.

Experimental

All experiments were carried out with our self-designed 3D vibrator shown schematically in Fig. 1, which vibrates in three orthogonal directions with different amplitudes (A) and frequencies (ω). This unit was successfully used in our previous experiments on the packing densification of spherical and non-spherical particles and powders (An et al., 2009; Li et al., 2011; An & Li, 2013; An, Xing, & Jia, 2014; An, He, Feng, & Qian, 2015). In the present work, the vibration amplitude (A) and duration time (t) were fixed at 0.5 mm and 300 s, respectively, and the vibration frequency was varied to ascertain its effect on the binary packing density. For simplification, the frequencies were the same in each of the three directions. Steel ball bearings of different sizes were used as the spheres (particle diameter, $d = 0.98$, 4.96, 7.50, 10.00, and 14.99 mm; theoretical density, $\rho_t = 7.761$ g/cm^3). To study the effect of the container size, five different-sized cylindrical PMMA containers (inner diameter, $D = 79.42$, 109.90, 140.38, 185.77, and 229.70 mm) were used.

Before vibration, a certain quantity (composition) of large and small spheres was weighted and mixed manually to obtain a uniform binary mixture. This was then gently poured (total feeding) into the container that was mounted on the vibration desk to form the initial packing. The packing density (also called the fractional density, defined as $\rho = V_p/V_c$, where V_p is the volume occupied by the spheres and V_c is the volume occupied by solids bed in the container) was then calculated by averaging the packing heights measured at different positions in the container to reduce the experimental error from the inclination of the top surface of the packing. Subsequently, densification of the mixture was conducted under the preset vibration conditions and the packing density was remeasured. Note that each packing density obtained was the average of the values from three repeated experiments to minimize the error. During vibration packing, the vibration conditions were controlled carefully to avoid size segregation and to realize dense random packings of the binary sphere mixtures.

Four parameters were considered during packing densification of the binary spheres. Each parameter was varied within the range given in Table 1. To identify the effects of each parameter, the other parameters were fixed while the parameter under investigation was varied. Therefore, a series of packing densities was obtained for comparison, with each value corresponding to a certain set of parameters.

Results and discussion

The effects of vibration frequency on the packing density of the binary sphere mixtures in different-sized containers were first studied. The contour map of the packing density distribution is shown in Fig. 2(a), where the volume fraction of large spheres and the size ratio were fixed at $X_L = 0.7$ and $r = 0.0654$, respectively. It can be seen that for each vibration frequency, the larger the container size, the higher the packing density. Moreover, for each container size, a higher packing density corresponds to an intermediate frequency; that is, a frequency that is either too large or too low is not beneficial to achieving an efficient packing densification. Therefore, to achieve a higher packing density, a larger container should be chosen and the vibration frequency should be properly controlled. In the following discussion, unless otherwise stated, the results were obtained from the largest container ($D = 229.70$ mm) to minimize the wall effects, which will be discussed later. To identify the frequency effects more clearly, we studied the ρ–ω relationship at various values of X_L and r, as plotted in Fig. 2(b) and (c),

Table 1
Parameter variation range in the experiments.

Container diameter, D (mm)	79.42, 109.90, 140.38, 185.77, 229.70
Particle size ratio, $r = d_s/d_L$	0.0654, 0.0980, 0.1307, 0.1975
Volume fraction of large spheres, X_L	0, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0
Vibration frequency, ω (rad/s)	50, 90, 110, 120, 130, 140