Particuology xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

Particuology

journal homepage: www.elsevier.com/locate/partic

Capillary bridges and capillary forces between two axisymmetric power–law particles

Lefeng Wang, Fengting Su, Huichao Xu, Weibin Rong, Hui Xie*

State Key Laboratory of Robotics and System, Harbin Institute of Technology, Harbin 150080, China

ARTICLE INFO

Article history:
Received 23 April 2015
Received in revised form 21 July 2015
Accepted 18 August 2015
Available online xxx

Keywords:
Capillary force
Power–law profile particles
Power–law index
Rupture distance

ABSTRACT

Capillary interactions are fundamentally important in many scientific and industrial fields. However, most existing models of the capillary bridges and capillary forces between two solids with a mediated liquid, are based on extremely simple geometrical configurations, such as sphere–plate, sphere–sphere, and plate–plate. The capillary bridge and capillary force between two axisymmetric power–law profile particles with a mediated constant-volume liquid are investigated in this study. A dimensionless method is adopted to calculate the capillary bridge shape between two power–law profile particles based on the Young–Laplace equation. The critical rupture criterion of the liquid bridge is shown in four forms that produce consistent results. It was found that the dimensionless rupture distance changes little when the shape index is larger than 2. The results show that the power–law index has a significant influence on the capillary force between two power–law particles. This is directly attributed to the different shape profiles of power–law particles with different indices. Effects of various other parameters such as ratio of the particle equivalent radii, liquid contact angle, liquid volume, and interparticle distance on the capillary force between two power–law particles are also examined.

Introduction

Capillary interactions play a significant role in many scientific and industrial fields. These interactions must be accounted for in studies of particle cohesiveness and wet agglomeration processes, which are related to soil mechanics, granular materials, pharmaceuticals, and civil engineering (Rossetti, Pepin, & Simons, 2003; Sun, Wang, & Hu, 2009). In recent years, capillary interactions between micro- and nano-particles have attracted much attention because of the rapid development of micro and nanotechnology. Capillary interactions can, in principle, be exploited for gripping applications, and several types of capillary grippers have been proposed for microparticle manipulation (Lambert, Seigneur, Koelemeijer, & Jacot, 2006; Fantoni, Hansen, & Santochi, 2013; Fan, Wang, Rong, & Sun, 2015). Capillary forces are also utilized to control self-assembly processes from the millimeter to nanometer scale in microsystem engineering (Mastrangeli et al., 2009). Moreover, capillary interactions greatly affect force measurements and nanomanipulation processes, as revealed in atomic force microscopy (AFM) studies (Butt, Cappella, & Kappl, 2005; Tabor, Grieser, Dagastine, & Chan, 2012; Kim, Shafiei, Ratchford, & Li, 2011; Onal, Ozcan, & Sitti, 2011). To use them effectively in these related fields, it is necessary to fully understand and accurately model capillary interactions.

There has been much work modeling and measuring capillary forces between two particles. Based on thermodynamic equilibrium and non-equilibrium conditions, two main cases are considered when modeling the capillary forces. The first case refers to a liquid bridge that has a constant curvature radius, which is determined from the unsaturated vapor pressure in the environment, as described by the Kelvin equation (Butt, 2008). Pakarinen et al. (2005) investigated the capillary force between a nanosphere and a flat surface and found that the often-used model of the humidity-independent capillary force is reasonable for spherical particles above 1 μm. Xiao and Qian (2000) measured the adhesion force between a Si_3N_4 AFM tip and SiO_2 or *n*-octadecyltrimethoxysilane (OTE)/SiO_2 substrates. For the former, the adhesion force first increases and then decreases because of strong capillary condensation. For the latter, the adhesive force is almost independent of humidity because of weak capillary condensation.

The second case corresponds to a constant-volume liquid bridge based on the energy principle. Rabinovich, Esayanur, and Moudgil (2005) calculated the capillary force between two spheres with a

* Corresponding author. Tel.: +86 45186404422.
E-mail address: xiehui@hit.edu.cn (H. Xie).

http://dx.doi.org/10.1016/j.partic.2015.08.005

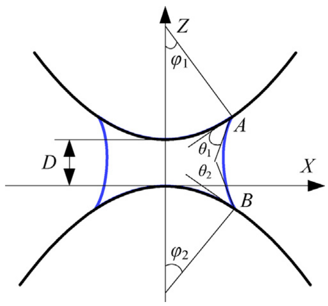

Fig. 1. Liquid bridge between two axisymmetric power–law profile bodies. The angle φ_1 is determined by the normal line of the particle profile at point A and the axisymmetric axis of the particle.

fixed volume liquid bridge, and discussed the applicability of the Derjaguin approximation for the interaction between the spheres. De Souza, Brinkmann, Mohrdieck, Crosby, and Arzt (2008) investigated the capillary force between two plates with chemically different properties, and noted that the capillary force decreases as the asymmetry in contact angles is increased with the fixed sum of contact angles. Yang, Tu, and Fang (2010) proposed a rupture model of a capillary bridge between a micro sphere and a plane, and it was found that the rupture distance increases with increasing spherical radius, sphere hydrophobicity, and environmental humidity.

Geometrical configurations considered in capillary force modeling have been mostly limited to sphere–plate (Israelachvili, 1992), sphere–sphere (Willett, Adams, Johnson, & Seville, 2000; Lian, Thornton, & Adams, 1993; Payam & Fathipour, 2011), and plate–plate (De Souza et al., 2008). However, in many real cases, particle shapes are not so ideal and simple. For example, the general shape of an AFM probe tip could be better described by a power–law axisymmetric function as a result of the fabrication method or in-use wear (Grierson, Liu, Carpick, & Turner, 2013). Recently, the capillary force between a power–law probe tip and a spherical particle or a plate under different humidity conditions was investigated (Wang & Régnier, 2015). The effects of relative humidity on the capillary forces were revealed in a dimensionless form which is applicable for equilibrium conditions of the liquid bridge. However, in applications such as micro/nano manipulation, we are more interested in the capillary force between two particles with a constant-volume liquid bridge.

In this study, the capillary bridge and capillary force between two power–law profile particles with a constant-volume liquid bridge are investigated. In the Modeling the capillary bridge and capillary force section, modeling processes of the capillary bridge and capillary force between two power–law particles are presented in detail. In the Results and discussion section, the rupture criteria for the liquid bridges are demonstrated, and effects of each parameter on the capillary forces are analyzed. Finally, conclusions are given in the Conclusions section.

Modeling the capillary bridge and capillary force

Fig. 1 shows the liquid bridge between two power–law profile bodies. In this figure, θ_1 and θ_2 are the liquid contact angles at the two solid–liquid interfaces. Herein, relatively small liquid bridges are considered and gravitational deformations of the menisci are neglected. For the liquid bridge between two equal spheres with radii R, it was found that a modified Bond number V^*B_o can be used to predict the effect of gravity on the pendular liquid bridge, in which V^* is the dimensionless volume with $V^* = V/R^3$, and B_o is the Bond number with $B_o = \rho g R^2/\gamma$ (Adams, Johnson, Seville, & Willett, 2002). The parameter ρ is the density of the liquid, g is the acceleration due to gravity, and γ is the surface tension of the liquid. When $V^*B_o < 0.01$, the effect of gravity can be neglected (Adams et al., 2002). It is assumed that this criterion is also applicable to the liquid bridge between two power–law profile particles.

The profile of the top particle can be written as

$$Z_1 = \frac{X^{n_1}}{n_1 R_1^{n_1-1}} + D, \tag{1}$$

where n_1 is the power–law shape index of the top particle. For special cases, it denotes a parabolic shape when $n_1 = 2$. R_1 can be treated as the equivalent radius of the top particle, and the equivalent half-filling angle can be defined as $\varphi_1 = X_A/R_1$ accordingly. D is the distance between the two power–law profile particles. When $n_1 = 1$, it is a conical shape with conical angle of 45° (Zheng & Yu, 2007). Similarly, the profile of the bottom particle can be written as

$$Z_2 = -\frac{X^{n_2}}{n_2 R_2^{n_2-1}}, \tag{2}$$

where n_2 is the power–law shape index of the bottom particle. R_2 can be treated as the equivalent radius of the bottom particle, and the equivalent half-filling angle can be defined as $\varphi_2 = X_B/R_2$ accordingly.

The dimensionless method was used to simplify the calculation of the liquid bridge shape. If we let $z = Z/R_1$, $x = X/R_1$, and $d = D/R_1$, then Eqs. (1) and (2) can be nondimensionalized as follows:

$$z_1 = \frac{x^{n_1}}{n_1} + d, \tag{3}$$

$$z_2 = -\frac{x^{n_2}}{n_2 \left(R_2/R_1\right)^{n_2-1}}. \tag{4}$$

According to the Young–Laplace equation, the liquid meniscus $Z_3 = f(X)$ satisfies the following equation:

$$-\frac{X''}{\left(1+X'^2\right)^{3/2}} + \frac{1}{X\left(1+X'^2\right)^{1/2}} = \frac{\Delta P}{\gamma}, \tag{5}$$

where $X' = \mathrm{d}X/\mathrm{d}Z$, $X'' = \mathrm{d}^2X/\mathrm{d}Z^2$. ΔP is the hydrostatic pressure difference across the air–liquid interface and γ is the surface tension of the liquid. Eq. (5) can be nondimensionalized as

$$-\frac{x''}{\left(1+x'^2\right)^{3/2}} + \frac{1}{x\left(1+x'^2\right)^{1/2}} = \frac{R_1 \Delta P}{\gamma} = \Delta p. \tag{6}$$

The dimensionless volume v of the liquid bridge is

$$v = v_0 - v_t - v_b, \tag{7}$$

where v_0, v_t, and v_b represent the dimensionless volume of the column formed by the meniscus profile, the dimensionless volume of the immersed part of the top power–law particle, and the dimensionless volume of the immersed part of the bottom power–law particle, respectively. These terms can be obtained through integration:

$$v_0 = \int_{z_B}^{z_A} \pi x^2 \mathrm{d}z_3, \tag{8}$$

$$v_t = \int_{d}^{z_A} \pi x^2 \mathrm{d}z_1 = \frac{\pi}{n_1+2} x_A^{n_1+2}, \tag{9}$$

$$v_b = \int_{z_B}^{0} \pi x^2 \mathrm{d}z_2 = \frac{\pi}{n_2+2} \left(\frac{R_2}{R_1}\right)^{n_2-1} x_B^{n_2+2} \tag{10}$$