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Efficient approaches of determining the motion of a spherical particle in a swirling fluid flow using weighted residual methods

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ABSTRACT

The motion of a spherical particle released in a swirling fluid flow is studied employing the least-squares method and method of moments. The governing equations are obtained and solved employing the two methods. The accuracy of the results is examined against the results of a fourth-order Runge–Kutta numerical method. The effects of various parameters, namely the initial radius, initial radial velocity, initial angular velocity, and drag-to-inertia ratio, on the non-dimensional velocity profiles and particle position distribution are considered. The results show that the radial velocity increases over time while the angular velocity decreases, and that an increase in the initial radial velocity increases the particle radial distance and angular velocity but decreases the radial velocity profile.

Introduction

The dynamics of a particle embedded in a fluid flow has gained much attention over the last two decades. The flow of a solid–liquid mixture has many industrial and engineering applications such as propulsion, slurry transportation, and transportation of food grains (Pagalthivarthi & Gupta, 2008).

Many researchers have investigated the problem of the motion of a particle settling under the effect of gravity. The solution has been derived only for a particle having a Reynolds number less than 1 (Basset, 1888; Oseen, 1927). Moreover, many researchers have studied the rotation of a small sphere suspended in a fluid vortex. Some have studied mass transport by a vortex ring (Domon, Ishihara, & Watanabe, 2000), while others have conducted vortex simulations for the interaction between a vortex ring and solid particles (Gan, Dawson, & Nickels, 2012; Uchiyama & Yagami, 2007). Kollmann (2011) applied a numerical method in the investigation of the flow generated by two swirling vortex rings. Their results show that the rings merge and form a single ring with complex internal structure.

The word "vortex" is reserved for models of swirling flow having one of three forms: the forced vortex, free vortex, and free spiral vortex. A vortex occurs in fluid flow because of the effects of pressure variations and shearing flows. In the case of the free vortex, because of the absence of external forces, the vortex usually evolves fairly quickly toward an irrotational flow pattern. For a free vortex, the radial velocity is zero and the tangential component of the particle velocity is inversely proportional to the distance r (radial position). A forced vortex has non-zero vorticity away from the core and can be maintained indefinitely in that state only through the application of an additional force that is not generated by the fluid motion itself.

Swirl phenomena have been examined computationally and experimentally. Computational fluid dynamics (CFD) method has been widely used to model the motion of a particle in a fluid vortex and swirling flow field. Although the CFD method provides results that are good and reliable, the results depend on the mesh density. Therefore, analytical methods provide useful results and are used instead of numerical methods to solve nonlinear differential equations.

Jalaal and Ganji (2010) studied the unsteady motion of a spherical particle falling in an incompressible Newtonian fluid. They derived analytical expressions for the velocity and acceleration of the particle employing the homotopy perturbation method (HPM). Torabi and Yaghoobi (2011) provided a novel solution, which was a combination of the HPM and Padé approximant, to improve the solution given by Jalaal, Ganji, and Ahmadi (2010). Yaghoobi and Torabi (2012) examined the unsteady motion of a non-spherical

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particle falling in a Newtonian fluid employing the differential transformation method (DTM). Jalaal and Ganji (2011) investigated the acceleration of a spherical particle rolling down an inclined tube filled with incompressible Newtonian fluid employing the HPM. As an important outcome, they found that the inclination angle does not affect the acceleration. Hatami and Domairry (2014) studied the equation of motion of a soluble particle falling in a Newtonian medium employing the Padé approximation of DTM (DTM-Padé). They considered that the mass of the particle was reduced by the solubility of the particle in the fluid, and the diameter of the particle therefore reduced linearly. Ghasemi, Palandi, Hatami, and Ganji (2012) discussed the convergency and accuracy of the variational iteration method and Adomian decomposition method for solving the motion of a spherical particle in Couette fluid flow. Hatami, Sheikholeslami, and Domairry (2014a) provided a novel solution, referred to as the multi-step differential transformation method (Ms-DTM), for spherical particles in plane Couette fluid flow and compared their results with previous numerical and analytical results. Hatami and Ganji (2014b) examined the motion of a spherical particle on a rotating parabola using the Ms-DTM. Recently, Hatami and Ganji (2014a) investigated the motion of a particle suspended in a fluid vortex employing the differential quadrature method and DTM with the Padé approximation. The above analytical methods have been demonstrated to be precise and efficient in solving a wide range of engineering problems (Ghasemi, Hatami, & Ganji, 2013; Ghasemi, Hatami, & Ganji, 2014b; Ghasemi, Zolfagharian, & Ganji, 2014c).

Motivated by the above-mentioned work, the aim of this study is to obtain the approximate solution for the motion of a particle in a fluid forced vortex. To solve the governing equations of the problem, we used two analytical methods: the least-squares method (LSM) and method of moments (MM). Additionally, series solutions for the particle position and velocity profiles were obtained, and the effects of different parameters on the radial position, tangential velocity and radial velocity of the particle were studied and shown graphically.

Problem description

We considered polar coordinates **r**–**θ** with the origin $r=0$. It is assumed that the speed of rotation of the fluid is approximately constant, and the study thus presents two-dimensional models for a particle in a free or forced vortex. The mass of the fluid rotates in the counter clockwise direction around the origin (see Fig. 1). As explained above, for the free vortex, the tangential velocity of the particle is inversely proportional to the distance r (i.e., $u_\theta = c/r$), while for the forced vortex, the tangential component of the particle velocity is $u_\theta = \Omega r$. Additionally, the radial velocity is zero in both cases. A spherical particle falls through the fluid at an initial radius of r_0, radial velocity of u_0, and zero angular velocity, at the instant t_0.

The drag force (F_D) that acts on and rotates the spherical particle within the flow can be written as

$$F_D = \frac{1}{2} c_D \rho u^2 A, \tag{1}$$

where c_D denotes the drag coefficient, which is a function of the Reynolds number, and ρ, u, and A are the fluid density, the relative velocity between the particle and fluid, and the projected area of the

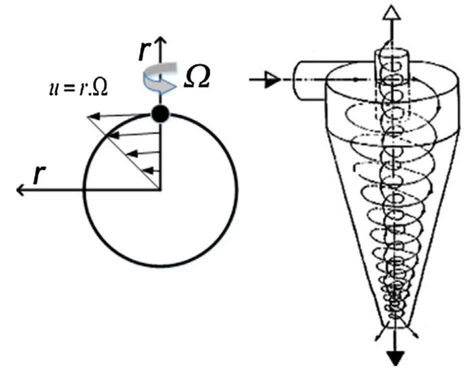

Fig. 1. Schematic of the motion of a spherical particle in an example of swirling fluid flow (filtering device).

sphere, respectively. According to Newton's second law, the equations of motion of the particle in the radial and tangential directions can be written as follows:

$$\text{Radial direction}: m\left(\ddot r - r\dot\theta^2\right) = -c_D A \frac{1}{2}\rho \dot r^2. \tag{2}$$

$$\text{Tangential direction}: \begin{cases} m\left(r\ddot\theta + 2\dot r\dot\theta\right) = c_D A \frac{1}{2}\rho\left(r\dot\theta - \frac{c}{r}\right)^2 & \text{for free vortex} \\ m\left(r\ddot\theta + 2\dot r\dot\theta\right) = c_D A \frac{1}{2}\rho\left(r\dot\theta - \Omega r\right)^2 & \text{for forced vortex} \end{cases}. \tag{3}$$

By introducing the following non-dimensional parameters

$$\Omega = \frac{\omega}{\omega_0}, \quad U = \frac{u}{u_0}, \quad R = \frac{r}{r_0}, \quad \tau = t\omega_0, \tag{4}$$

the equations of the motion of the particle can be written in non-dimensionalized form as (El-Naggar & Kholeif, 2012)

$$\begin{cases} \dot R(\tau) = \frac{u_0}{r_0\Omega}U(\tau) \\ \frac{u_0}{r_0\Omega}\dot U(\tau) - R(\tau)\bar\omega(\tau)^2 = -\alpha^2\left(\frac{u_0}{r_0\Omega}\right)^2 U(\tau)^2 \\ R\dot\Omega(\tau) + \frac{u_0}{r_0\omega_0}2U(\tau)\Omega(\tau) = \alpha^2 R^2 \begin{cases} (\Omega-1)^2 & \text{Forced vortex} \\ \left(\Omega - \frac{1}{R^2}\right)^2 & \text{Free vortex} \end{cases} \end{cases}, \tag{5}$$

where the non-dimensional parameter $\alpha^2 = \left(1/2 c_D A \rho r_0\right)/m$ is the drag-to-inertia ratio. The initial conditions for solving the governing equations are

$$U(0) = 1, \quad R(0) = 1, \quad \Omega(0) = 0.$$

Principles of the LSM and MM

The LSM and MM are two examples of approximation techniques called weighted residual methods (WRMs) that are used to solve differential equations. Recently, the MM was applied by Lemos, Secchi, and Biscaia (2014) to advection–diffusion problems with chemical reactions to solve boundary value problems based on polynomial approximations. Other engineering problems have also been analyzed using the MM (Falola, Borissova, & Wang, 2013; Ortiz-Mora, Diaz, Gómez-Alcalá, & Dengra, 2013). The LSM and MM have many advantages over other analytical and numerical methods and are used to solve nonlinear differential equations. Some of these advantages are listed as follows (Ghasemi, Hatami,

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