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# Investigation of the characteristics of particulate flows through fibrous filters using the lattice Boltzmann method

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ABSTRACT

A fibrous filter is one of the most common systems used to separate suspended particles from air. Two important factors (i.e., the pressure drop and capture efficiency) are usually used to evaluate the performance of this type of filter. This study considers three two-dimensional arrangements of fibers (parallel, staggered, and random) to geometrically model fibrous media. The lattice Boltzmann method is employed to numerically simulate fluid flow through the filter. The Lagrangian form of the equation of motion of a particle is numerically solved to track the path of each particle in the flow field, where a one-way interaction between the fluid and particles is considered. The effects of pertinent parameters such as the fiber arrangement, solid volume fraction, particle-to-fiber diameter ratio, particle-to-fluid density ratio, Reynolds number, Stokes number, and size of the fibrous medium on the pressure drop and capture efficiency are studied. The obtained results are compared with existing empirical and theoretical findings and discussed.



## Introduction

Aerosols that are produced either by nature or through human activities (e.g., the activities of the mining and cement industries) can pose serious threats to human health. Additionally, many industries, such as the industry of power generation, require clean incoming air to operate efficiently. One of the air purification methods currently available is the use of highly efficient fibrous filters. Since the beginning of the last century, scientists have attempted to develop mathematical formulations and have carried out experimental investigations to predict and improve the performance of fibrous filters. More complete reviews on filtration can be found in the literature (Brown, 1993; Spurny, 1998; Tien, 2012).

In designing highly efficient fibrous filters, two important factors, namely the pressure drop and capture efficiency, should be considered. There are several analytical, numerical, and empirical correlations that can be used to determine the pressure drop (Davies, 1973; Jackson & James, 1986; Kuwabara, 1959) and capture efficiency (Brown, 1993; Lee & Gieseke, 1980; Lee & Liu, 1982) of fibrous filters. The principles of fibrous filtration theory are based on the work of Kuwabara (1959), who numerically and analytically simulated the fluid flow in a unit cell containing a single fiber while considering periodic boundary conditions. In this cell model, two-dimensional geometries of the fibrous filters are simplified by placing perfectly clean fibers in square or hexagonal arrangements, perpendicular to the flow direction. One of the limitations of the cell model (Kirsch, 2007; Liu & Wang, 1997) is that it is only applicable to Brownian particles. Therefore, efforts have been made to model fibrous media as an array of fibers with inline or staggered (ordered) arrangement (Chen, Cheung, Chan, & Zhu, 2002; Kirsch, 2007; Liu & Wang, 1996, 1997; Przekop, Moskal, & Gradon, 2003; Rao & Faghri, 1988), which can be used to filter particles having higher inertia.

Owing to the effect of the fiber arrangement on the pressure drop and capture efficiency (Liu & Wang, 1997), more studies on modeling the geometry of fibrous media are needed, especially studies on the random arrangement, which more closely resembles the real arrangement of fibers. In recent years, fibrous filters have been simulated in a series of works, mostly using Fluent software. These studies have generally investigated the effects of various filter conditions on the filter performance (Fotovati, Vahedi Tafreshi,

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& Pourdeyhimi, 2010; Hosseini & Vahedi Tafreshi, 2010a,b; Vahedi Tafreshi, A Rahman, Jaganathan, Wang, & Pourdeyhimi, 2009; Wang, Maze, Vahedi Tafreshi, & Pourdeyhimi, 2006; Xu, Wu, & Cui, 2014).

Given that it is important to use filters in places where humans live and work, and to improve the performance and environmental compliance of industries, this paper investigates the effects of pertinent parameters on the performance of fibrous filters. A common approach is to use an ordered array of cylinders as the arrangement of fibers in a filter medium. However, the suitability of such geometry for this purpose has been little investigated in the available literature. Therefore, to clarify the effect of different arrangements of fibers on the filter performance, this study compares results obtained for two regular fiber arrangements (in line and staggered) and a random arrangement. Since the geometries of fibrous filters are rather complicated, the lattice Boltzmann method (LBM) is used, which has been shown to be a robust and promising tool for simulating fluid flows in complex geometries (Chen & Doolen, 1998). To analyze the performance of the fibrous filter, the effects of pertinent parameters such as the fiber arrangement, solid volume fraction (designated SVF or $\alpha$), particle-to-fiber diameter ratio, particle-to-fluid density ratio, fluid flow face velocity (inlet average velocity), and filter length on the pressure drop and capture efficiency are investigated. The dependency of the capture efficiency on the filter length for different fiber arrangements and particle sizes is studied here, which has been sparsely investigated in previous works.

## Governing equations and numerical method

### *Fluid flow*

Fluid flow through an array of fibers is investigated using the LBM. The evolution of the density distribution function $f\left(\vec{r}, \vec{e}, t\right)$ can be evaluated using the Boltzmann transport equation; however, this equation is difficult to solve because of its complicated collision term. Bhatnagar, Gross and Krook (1954) presented a simplified model, known as the BGK approximation, to replace this complicated collision operator. By discretizing the Boltzmann equation with the BGK approximation with respect to the microscopic velocity space, the lattice Boltzmann equation takes the form (Sukop & Thorne, 2007)

$$\frac{\partial f_i\left(\vec{r}, t\right)}{\partial t} + \vec{e}_i \cdot \vec{\nabla} f_i\left(\vec{r}, t\right) = \frac{f_i\left(\vec{r}, t\right) - f_i^{\text{eq}}\left(\vec{r}, t\right)}{\tau}, \quad (1)$$

where $f^{\text{eq}}$ is the equilibrium distribution function (the Maxwell–Boltzmann distribution function),$\tau$ is the relaxation time, and $e_i$ is the discrete velocity. Index $i$ represents the lattice direction and runs from 0 to 8 for the most current two-dimensional lattice (known as the $D_2Q_9$ model), with zero indicating the rest state, or zero microscopic velocity.

Considering the lattice speed $c = \delta x/\delta t$, where $\delta x$ is the spacing between lattice nodes and $\delta t$ is the time step, the magnitude of $e_i$ equals $\sqrt{2}c$ for diagonal vectors and equals $c$ for all other vectors (except for $e_0$, which is zero). The speed of sound is $c_s = c/\sqrt{3}$. The equilibrium distribution functions are specified in such a way that the Navier–Stokes equations can be recovered through Chapman–Enskog expansion (Chen & Doolen, 1998). The relaxation time and Maxwell–Boltzmann distribution function used in the $D_2Q_9$ model, considering $\delta x = \delta t = 1$, are (Succi, 2001)

$$\tau = \frac{1}{2} + 3\theta, \quad (2)$$

$$f_i^{\text{eq}} = \rho\omega_i\left[1 + 3\vec{e}_i \cdot \vec{u} + 4.5\left(\vec{e}_i \cdot \vec{u}\right)^2 - 1.5\vec{u} \cdot \vec{u}\right], \quad (3)$$

where $\omega_i$ denotes constant weights, the values of which depend on the particular discretization model. The weights for the $D_2Q_9$ model are $\omega_0 = 4/9$ and $\omega_i = 1/9$ for $i = 1, 2, 3, 4$ and $\omega_i = 1/36$ for $i = 5, 6, 7, 8$. The lattice Boltzmann equation can be solved in two steps (Sukop & Thorne, 2007). The first is the collision step given by

$$\tilde{f}_i\left(\vec{r}, t + \delta t\right) = f_i + \frac{f_i^{\text{eq}} - f_i}{\tau}, \quad (4)$$

and the second is the propagation step given by

$$f_i\left(\vec{r} + \vec{e}_i\delta t, t + \delta t\right) = \tilde{f}_i\left(\vec{r}, t + \delta t\right), \quad (5)$$

where $\tilde{f}_i$ and $f_i$ are the post-collision and pre-collision density distribution functions, respectively.

Macroscopic flow parameters such as pressure and velocity are evaluated according to

$$P = \frac{1}{3}\sum_{i=0}^{8} f_i, \quad (6)$$

$$\vec{u} = \frac{1}{3P}\sum_{i=0}^{8} f_i\vec{e}_i. \quad (7)$$

For more information, see Luo, Krafczyk, and Shyy (2010).

In this paper, the concentration of particles in the flow field is considered low enough that the assumption of the one-way interaction between particles and fluid is justifiable. It should be noted that the particle effects on the flow field can be ignored in the one-way interaction approach.

### *Motion of particles*

In particulate flows, the track of each particle is determined by solving the Lagrangian equation of motion of the particle:

$$m_{\text{p}}\frac{\text{d}\vec{u}_{\text{p}}}{\text{d}t} = \sum \vec{F}. \quad (8)$$

In the above equation, the subscript $p$ indicates the parameters related to the particles. For particles in the diameter range of 0.01–20 μm, hydrodynamic forces such as the Basset force and the virtual mass are extremely small (Li & Ahmadi, 1992) and can be ignored. Additionally, Brownian diffusion is negligible for particles larger than 1.0 μm (Friedlander, 1977). Since the gravitational force is proportional to the cube of the particle radius, it can generally be ignored for very small particles (Lantermann & Hänel, 2007). Therefore, in the present study, the drag force is the only force that is considered to be exerted on the particles. At low Reynolds numbers, as considered here, this force can be derived using the Stokes law (Friedlander, 1977):

$$F_D = \frac{3\pi\mu\left(u - u_p\right)d_p}{C_c}, \quad (9)$$

where $C_c$ is the Cunningham correction factor (Friedlander, 1977).

Following the Lagrangian approach, the position components ($x_p$ and $y_p$) and velocity components ($u_p$ and $v_p$) of the particles at each time step need to be determined. The components of the particle velocity at time $t$ can be determined using numerical methods such as the fourth-order Runge–Kutta method to solve Eq. (8). The initial velocity of the particle is equal to the inlet fluid velocity. The fluid velocity at a particle's position is assumed to be constant during the integration time step. The position of each particle can then be calculated by numerically solving two initial-value problems:

$$\frac{\text{d}x_p}{\text{d}t} = u_p, \quad (10)$$