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# A statistical approach to scaling size effect on strength of concrete incorporating spatial distribution of flaws

Wei-Sheng Lei [a,*], Zhishui Yu [b]

[a] Currently at: Applied Materials, Inc., Bldg. 81, M/S 81305, 974 East Arques Ave., Sunnyvale, CA 94085, USA
[b] School of Materials Engineering, Shanghai University of Engineering Science, 333 Long Teng Rd., Shanghai 201620, China

## HIGHLIGHTS

- Two composite parameters are adopted for size scaling of the strength of concrete.
- The parameters include failure probability and the volume of fracture process zone.
- They are validated in proportional scaling with 3 sets of published strength data.

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ABSTRACT

The spatial distribution of flaws in a solid has a direct impact on the cumulative probability of failure due to brittle fracture. Accordingly, two composite parameters incorporating the cumulative probability of failure and the volume of fracture process zone are identified and adopted to characterize the size effect on the strength of concrete. Instead of being pre-assumed a specific function, the cumulative distribution function of fracture strength, namely the cumulative probability of fracture, is inferred for either the Poisson or the uniform spatial distributions of flaws from the synchronized analysis of multiple strength data sets measured from different sized specimens of geometrical similarity under a same loading mode (proportional scaling). This approach is validated for the case of proportional scaling by evaluating three representative sets of published strength data of concrete from uniaxial tension, uniaxial and equibiaxial flexure tests. Depending on the specific specimen size, the spatial flaw distribution may follow either the Poisson postulates or the uniform law, while the strength distribution of concrete does not necessarily always follow the Weibull statistics.



## 1. Introduction

The inverse correlation between the nominal strength of concrete structures and their geometrical dimensions is known as the size effect. The size effect presents a significantly difficult challenge to the transference of laboratory measured strength data of small-scale specimens to the design against failure of large-scale structures. Therefore, it has stimulated constant research efforts, with both statistical approaches (e.g. in [1]) and deterministic methods (e.g. in [2–5]) being adopted. An exhaustive review on this topic is out of the scope of this study. Instead, some basic aspects are emphasized here to better position the objective of this study:

(1) *Size scaling of strength.* The process of (either proportionally or non-proportionally) stretching or shrinking the dimension(s) of an object is known as size scaling. The ultimate purpose or expectation for studying the size effect is to extrapolate or transfer the strength data collected from laboratory based small-scale specimens to full-scale structural components. This can be partitioned into two tasks: First is the scalability of size effect on strength for a set of geometrically similar structures under a nominally same loading mode (proportional scaling). Second is the transferability of laboratory measured strength data from a small-scale specimen with simple geometry and loading mode to the performance of an arbitrary full-scale structure of usually much complex geometry and loading conditions (non-proportional scaling).

As an example for proportional scaling as the first scenario, Fig. 1 schematically illustrates an axisymmetrically generalized

* Corresponding author.
*E-mail addresses:* leiws2008@gmail.com (W.-S. Lei), Yu_Zhishui@163.com (Z. Yu).

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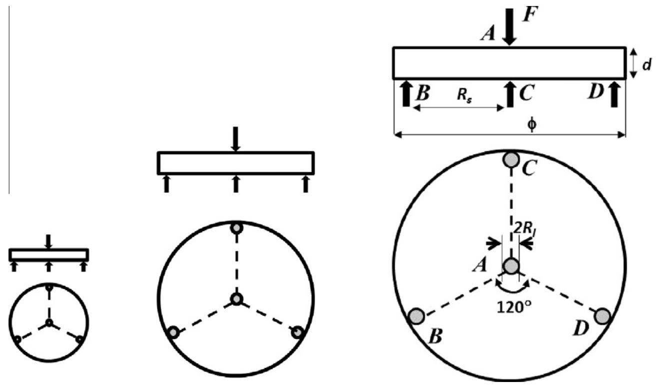


**Fig. 1.** Schematic illustration of ASTM C1550 test on a set of geometrically similar specimens.

three-point bending test of a group of circular plates with different sizes (named as small, middle and large specimens) according to ASTM C1550 [6], in which a circular plate is supported at three equally spaced points (B, C, D) along the circumference and loaded in the center point A. Here $\phi$ is plate diameter, *d* is plate thickness, *F* is the force applied to the center of the plate with the loading area of diameter $2R_l$ and the loading span radius $R_s$. For different specimens, the ratio of $d{:}R_l{:}R_s{:}\phi$ keeps the same to ensure their similarity with respect to geometry and loading mode.

In this case, one question naturally arises on the minimum specimen size that ensures the scalability of size effect on strength.

For non-proportional scaling as the second scenario, Fig. 2 schematically illustrates a small-sized prismatic beam of rectangular cross section in a four-point flexure setup and a large-sized round plate in a "ring-on-ring" flexure setup. In the four-point bending test, *d* and *b* are the thickness and width of the specimens, 2*c* and 2*l* are the inner and outer lading spans, respectively. $\sigma_p$ refers to the peak stress in the beam, and is often taken as the nominal strength. The stress state is uniaxial. In the ring-on-ring test, *d* and $\phi$ are plate thickness and diameter, $R_l$ and $R_s$ are the inner and outer (or support) loading span radius, respectively. The stress state is biaxial. In this case, it is of critical importance to understand whether it is feasible to transfer strength data from a small-sized specimen in a simple stress state to a real structure subjected to a much complicated stress state. If different stress states induce noticeable change win microcrack population and

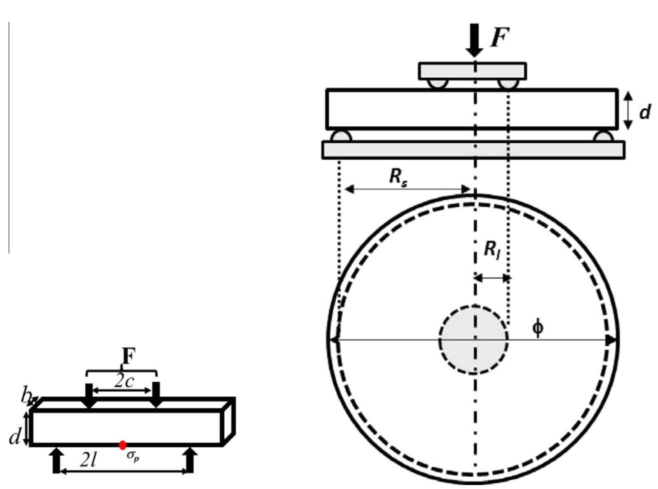


**Fig. 2.** Schematic illustration of a four-point flexure setup of a prismatic beam (left) and a ring-on-ring test of a round plate (right).

microscopic fracture criterion/mechanism, it may raise a fundamental difficulty for the strength extrapolation.

(2) *Inherent variation of strength.* In addition to the size dependence, the strength of concrete also exhibits a random dispersion. The dispersion of strength refers to the large scatter in the strength value measured from specimens of nominally identical geometrical dimensions under nominally same loading condition, due to the random distribution of defects inside concrete [6–9]. As a result, in reality, the size effect cannot be decoupled from the inherent dispersion of strength. This necessitates the statistical approach to evaluating the size effect on strength of concrete and brittle fracture in general.

(3) *The weakest link postulate vs. Weibull statistics.* Most statistical analysis of brittle fracture, e.g. [1,10–13], has been based on the weakest link postulate, which assumes that the strength of a structure is determined by its weakest volume element. In practice, the weakest link postulate based Weibull statistics [14,15] has been commonly adopted to describe the variation in the strength of materials:

$$P = 1 - exp\left[-\int_V \left(\frac{\sigma - \sigma_{th}}{\sigma_0}\right)^m \cdot \frac{\delta V}{V_0}\right] \quad (1)$$

or

$$P = 1 - exp\left[-\int_V \left(\frac{\sigma}{\sigma_0}\right)^m \frac{\delta V}{V_0}\right] \quad (2)$$

for $\sigma_{th} = 0$, where P is the cumulative probability of failure of a solid of volume V, $\delta$V is a differential volume, the quantity $1/V_0$ refers to the average number of microcracks per unit volume. In other words, $V_0$ is the average volume occupied by each microcrack, $\sigma_{th}$ is the threshold strength, $\sigma_0$ is the scale parameter, and m is the shape factor or Weibull modulus.

Obviously, the cumulative probability P given by Eqs. (1) and (2) is normalized for $\sigma_{th} \leqslant \sigma < \infty$ and $V \geqslant V_0$, with no need to assume $V/V_0 \to \infty$. The corresponding probability density function (PDF) in terms of fracture strength (S) is as follows:

$$g(S) = \frac{m}{\sigma_0} \cdot \left(\frac{S - \sigma_{th}}{\sigma_0}\right)^{m-1} \cdot \exp\left[-\left(\frac{S - \sigma_{th}}{\sigma_0}\right)^m\right] \quad (\sigma_{th} \leqslant S < \infty) \quad (3)$$

Bazant [1] addressed the difference between the infinite and the finite weakest-links, which is concretized by the volume (V) of a solid of interest and is determined by the ratio of the characteristic structural size to the size of a representative volume element (RVE) (typically equal to 2–3 grain or inhomogeneity size) to specify the applicable range of Weibull statistics. But since as just pointed out, the value of volume (V) itself or the length of a chain is not a factor to validate Weibull statistics. The physical change of fracture mechanism due to the change of volume (V) should be the root cause leading to strength deviation from Weibull statistics, as will be further discussed in Section 3. The Weibull statistics is also justified by the extreme value theory for risk analysis. According to Fisher and Tippet [16], Weibull distribution, Gumbel distribution, and Frechet distribution are the only three asymptotic forms of the extreme value distribution. However, mounting experimental evidence has suggested some drawbacks of the Weibull statistics for fracture of brittle materials [17–23], with alternative non-Weibull statistical models such as the Gumbel distribution [21] and the so-called Duxbury-Leath distribution [22,23] being used for characterizing strength dispersion of different materials. Therefore, instead of pre-assuming that the strength of concrete follows the Weibull distribution or any other specific statistical distribution, and then do data fitting for justification, it is more reasonable