Particuology 18 (2015) 35–41

Contents lists available at ScienceDirect

Particuology

journal homepage: www.elsevier.com/locate/partic

Revisiting rolling and sliding in two-dimensional discrete element models

Yucang Wang[a,b,*], Fernando Alonso-Marroquin[c], Sheng Xue[b], Jun Xie[b]

[a] School of Engineering and Technology, Central Queensland University, Australia
[b] Earth Science and Resource Engineering, The Commonwealth Scientific and Industrial Research Organisation (CSIRO), Australia
[c] School of Civil Engineering, The University of Sydney, Australia

ARTICLE INFO

Article history:
Received 3 March 2014
Received in revised form 15 April 2014
Accepted 16 April 2014

Keywords:
Discrete element method
Rolling
Sliding

ABSTRACT

It has long been recognized that the rotation of single particles plays a very important role in simulations of granular flow using the discrete element method (DEM). Many researchers have also pointed out that the effect of rolling resistance at the contact points should be taken into account in DEM simulations. However, even for the simplest case involving two-dimensional circular particles, there is no agreement on the best way to define rolling and sliding, and different definitions and calculations of rolling and sliding have been proposed. It has even been suggested that a unique rolling and sliding definition is not possible. In this paper we assess results from previous studies on rolling and sliding in discrete element models and find that some researchers have overlooked the effect of particles of different sizes. After considering the particle radius in the derivation of rolling velocity, all results reach the same outcome: a unique solution. We also present a clear and simple derivation and validate our result using cases of rolling. Such a decomposition of relative motion is *objective*, or independent of the reference frame in which the relative motion is measured.

Introduction

Granular materials consist of a large number of particles, each having translational and rotational motion depending on the total force and torque applied. Particles interact via contact areas and move relative to neighboring particles. The relative motion between touching particles includes motion in the normal direction, sliding in the tangential direction, and rolling over one another. Therefore, the macroscopic behavior of granular assemblies can be very complex. It has long been recognized that particle rotation and rolling play a key role in the mechanical behavior of granular materials, especially for those composed of circular or spherical particles. This has been pointed out since the pioneering work on rolling resistance by Oda, Konishi, and Nemat-Nasser (1982), who defined the rolling velocity between circular particles, and observed that inter-particle rolling dominates the microscale deformation of granular media.

The discrete element method (DEM) has emerged as an ideal tool to investigate the behavior of granular materials (Cundall & Strack, 1979). Different researchers reported that in DEM simulations of granular flow, both single-particle rotation and rolling resistance need to be incorporated into the model, otherwise unrealistic results are generated. Iwashita and Oda (1998, 2000) noted that the conventional DEM could not reproduce the large voids and high rotational gradients observed in shear band experiments. They recognized that rolling resistance causes an arching action at the contact points, and permits the easy formation of voids in physical tests. Therefore they proposed a modified model of the conventional discrete element method that takes the rolling resistance into account. Bardet and Proubet (1991) and Bardet (1994) examined the structure of shear bands in granular materials by simulating idealized granular media numerically. They showed that particle rotations concentrate inside shear bands and found that rotations have a significant effect on the shear strength of granular materials.

Tordesillas et al. (Tordesillas, Peters, & Muthuswamy, 2005; Tordesillas & Walsh, 2002) incorporated rolling resistance in the DEM and examined the influence of particle rotation and rolling resistance in the rigid flat-punch problem. They found that extensive particle rotations occur near the edges of the punch where

* Corresponding author at: School of Engineering and Technology, Central Queensland University, Australia. Tel.: +61 0749232576.
E-mail addresses: y.wang2@cqu.edu.au, yucang_wang@hotmail.com (Y. Wang).

http://dx.doi.org/10.1016/j.partic.2014.04.013
1674-2001/

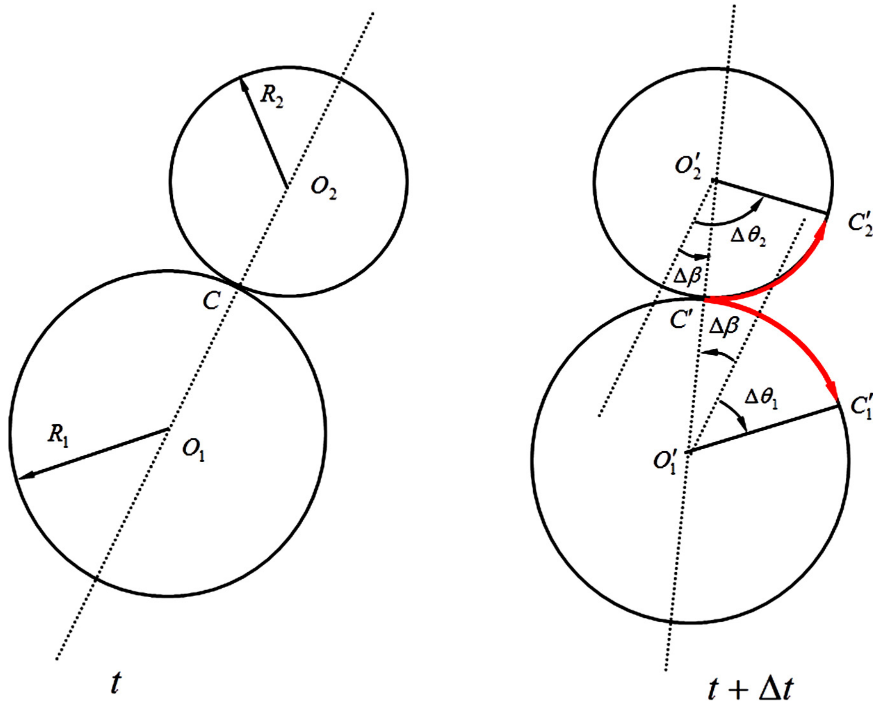

Fig. 1. Kinematic scheme of two disks in contact at times t and $t+\Delta t$.

high stress concentrations exist. These rotations lead to dilatation in the region adjacent to the sides of the punch. Wang and Mora (2008) showed that when only normal forces are transmitted, or when rolling resistance is absent, laboratory tests of wing-crack extension cannot be reproduced. The effect of rolling friction on granular flows has also been reported on recently by Balevicius, Sielamowicz, Mroz, and Kacianauskas (2012) and Goniva, Kloss, Deen, Kuipers, and Pirker (2012).

A quantitative investigation of the effects of rolling and sliding using the DEM demands a clear and unambiguous definition and calculation of rolling and sliding deformation. In principle, the relative motion between two particles in contact can be decomposed into several independent components: relative motion in the normal, tangential directions or sliding, relative rolling; and in the three-dimensional (3D) case, relative torsion. However, even for the simplest two-dimensional (2D) case involving circular particles, surprisingly, there is no agreement on the best way to define rolling and sliding. Different definitions and calculations of rolling and sliding have been proposed (Ai, Chen, Rotter, & Ooi, 2011; Alonso-Marroquin, Vardoulakis, Herrmann, Weatherley, & Mora, 2006; Bagi & Kuhn, 2004; Bardet, 1994; Bardet & Proubet, 1991; Iwashita & Oda, 1998; Jiang, Yu, & Harris, 2005; Kuhn & Bagi, 2004a,b; Luding, 2008; Mohamed & Gutierrez, 2010; Tordesillas et al., 2005; Tordesillas & Walsh, 2002). Some sources contradict other sources and this has led not only to confusion in the DEM field, but also to some researchers suggesting that there is no unique way to define the rolling displacement (Bagi & Kuhn, 2004).

The objective of our work is to answer three questions: Is there a unique way to define rolling and sliding deformation? If there is, how are rolling and sliding best determined in general cases? How can the different definitions of rolling resistance be consolidated in a unique formula? In this paper, we focus on the kinematics of two particles only. A thorough investigation of rolling resistance models is beyond the scope of this paper, but can be found in the literature (Ai et al., 2011; Mohamed & Gutierrez, 2010).

Problem statement

Fig. 1 shows the kinematic scheme of two discs in contact. During a time step from t to $t+\Delta t$, two particles 1 and 2, with radii R_1 and R_2, respectively, remain in contact. At time t, let O_1, O_2, and C denote the centers of the two particles and the contact point, respectively. At time $t+\Delta t$, the current centers of the two particles and the contact point are O'_1, O'_2, and C', respectively. The original contact point at C now appears at C'_1 on particle 1 and at C'_2 on particle 2. $\Delta\theta_1$ is the angle between O_1O_2 and $O'_1C'_1$, and $\Delta\theta_2$ is the angle between O_1O_2 and $O'_2C'_2$. By taking counter-clockwise rotation as positive, these two angles represent the incremental rotations of the two particles during the time step from t to $t+\Delta t$. $\Delta\beta$ is the angle between O_1O_2 and $O'_1O'_2$, representing an incremental change in the angle of contact direction between the particles. The arcs $C'C'_1$ and $C'C'_2$, which represent the displacement of the contact point C on particles 1 and 2, are denoted by Δa and Δb, respectively, and are positive when measured in a counter-clockwise direction. Generally, Δa and Δb have a rolling and sliding component, denoted by ΔU_r and ΔU_s, respectively. Now the question is how to determine and calculate ΔU_r and ΔU_s. To answer this question, we first need to define clearly what pure rolling and sliding are.

Definition of pure rolling and sliding

We start by defining rigid-body rotation (RBR). RBR occurs when two particles rotate together as a single rigid body. The distance between any arbitrary chosen points on the two particles remains constant during the motion. In this case (see Fig. 2), we have:

$$\Delta\beta = \Delta\theta_1 = \Delta\theta_2 \neq 0, \tag{1a}$$

$$\Delta a = \Delta b = 0, \tag{1b}$$

$$\Delta U_r = \Delta U_s = 0. \tag{1c}$$