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Role of particle stiffness and inter-particle sliding friction in milling of particles

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ABSTRACT

Discrete element method (DEM) has been used to investigate the effects of particle elastic modulus and coefficient of inter-particle sliding friction on milling of mineral particles. An autogeneous mill of 600 mm diameter and 320 mm length with 14,500 particles has been selected for the simulation. Various mill performance parameters, for example, particle trajectories, collision frequency, collision energy and mill power have been evaluated to understand the effects of particle elastic modulus and inter-particle sliding friction during milling of particles.

For the given model, it has been concluded that at high energy range, as the elastic modulus and particle sliding friction increase the energy dissipated among the particles increases. The collision frequency increases with the increase in elastic modulus, however, this trend is not clearly observed with increasing inter-particle sliding friction. The power draw of the mill increases with the increase in fraction of mill critical speed.

Introduction

Mineral particles possess a distribution of mechanical properties, which are governed by sizes, flaws, origins, and shapes of the particles, and accordingly the distribution dictates the performance of any processing equipment. In most cases, the same processing equipment is used to process mineral particles differentiated by physical properties. Such a trial method may lead to an inaccurate description of the processing equipment. In this regard, this paper explores the effect of particle elastic modulus and coefficient of inter-particle sliding friction in an autogeneous mill to understand their role in milling kinematics using the discrete element method (DEM). In addition, one of the mill operating parameters, called the fraction of critical speed, is also evaluated to study its effect on mill performance. The critical speed of a mill can be calculated with, $RPM = 42.3D^{-0.5}$, where RPM is the revolution per minute and D is the diameter of the mill in metre. This relationship is valid for most industrial mills where the ratio of mill diamter to ball diameter is essentially greater than 20. The formula is based on the assumption that there is no slip between the balls and the mill shell.

Particle elastic modulus, an indication of material stiffness, is defined as particle stress over strain which the particle undergoes in the elastic range. The particle stiffness is directly proportional to the elastic modulus for a given dimension of particles as described by the equation $k = YA/r$, where k is the stiffness constant, Y is the elastic modulus, A is the contact area, and r is the radius. Inter-particle sliding friction is the measure of resistance the particles experience when they come in contact with their near neighbours when they are in sliding motion.

Comminution is a well known energy intensive process and as a result, most of the investigations in this area are focused on understanding grinding mechanisms and refining milling circuits. The performance of mills is affected by many variables associated with mill configuration, grinding medium property and operational conditions. It has been proposed that up to 44 parameters may affect grinding performance in traditional grinding circuits although they are not equally important (Molls & Hornle, 1972). To find out the exact impact of these parameters, extensive studies have been carried out in the last decade or so (Becker & Schwedes, 1999; Berthiaux, Heitzmann, & Dodds, 1996; Blecher, Kwade, & Schwedes, 1996; Eskin, Zhupanska, Hamey, Moudgil, & Scarlett, 2005; Jayasundara, Yang, & Yu, 2012; Khanal & Morrison, 2008, 2009; Kwade, Blecher, & Schwedes, 1996; Stadler, Polke, Schwedes, & Vock, 1990; Theuerkauf & Schwedes, 1999; Vital et al., 2008).

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Most of these studies have been carried out to investigate the effect of grinding media properties on mill performance with only a limited investigation undertaken to understand the effect of particle stiffness and sliding friction on the process parameters (Khanal & Morrison, 2008, 2009). A similar study on effect of elastic modulus and sliding friction on particle packing conditions has been investigated by Antony and Krutt (2009).

Particles with a low shear modulus require more energy to achieve the same number of collisions as particles with a high modulus (Khanal & Morrison, 2008). For the same amount of collision energy, particles with a higher shear modulus generates more collisions than the particles with a lower modulus, suggesting stiff particles should be more energy efficient in generating fragments than the less stiff particles (Khanal & Morrison, 2008).

Mills operating at or below 75% of critical speed seem to be insensitive to the changes in particle coefficient of restitutions and friction but are sensitive to the changes in particle size distribution, however, 2%–6% variations have been observed in changing the charge material properties for higher mill rotation speeds (Cleary, 1998). In contrast, other researchers have shown that the power draw of the ball mills depends on the coefficient of friction (Mishra & Rajamani, 1992; Mishra & Thornton, 2001; Nierop, Glover, Hinde, & Moys, 2001).

In milling operations shear and normal forces (or stresses) are active however their ratio would depend on the milling and particle parameters, for example mill speed, stiffness, coefficient of sliding friction, cohesiveness of particles etc. It is expected that the higher coefficient of stiffness and inter-particle sliding friction tend to increase the number of contacts among particles and the abrasive mechanism.

Using DEM simulation the importance of friction between mineral particles and the mill shell have been highlighted by Khanal and Morrison (2008) and Mishra (2003). At a higher energy range, a larger coefficient of friction seems to produce a higher number of collisions but at the lower energy range the effect is still not clear (Khanal & Morrison, 2008). At higher speeds, a lower friction material is found to lead to higher power draw (Cleary, 1998). Mills rotating at lower speeds are suitable to abrade particles whereas the higher speed mills generate more fragments and are more suitable for incremental body breakage of particles (Khanal & Morrison, 2008, 2009).

The objective of this paper is to undertake a parametric investigation of the effects of particle elastic modulus and coefficient of inter-particle sliding friction in an autogeneous mill environment in order to understand their role in milling kinematics. The study is undertaken using the DEM. These two parameters have been investigated and results are compared at an identical milling condition. The paper also investigates the effect of critical speed on mill performance. The paper uses the collision frequency versus impact energy as an evaluation method to investigate the energy when particles collide.

DEM model

In DEM, particles are considered to be distinct elements, in this case spheres, and the laws of motion and material constitutive laws are applied to each element. The 3D DEM model used in this work is based on the soft-sphere model which has been extensively used to study various phenomena, such as particle packing, transport properties, hopper flow, mixing and granulation (Jayasundara, Yang, Yu, & Curry, 2006; Zhu, Zhou, Yang, & Yu, 2007). Due to the computational and model limitations this paper has considered only spherical particles in the simulations. The motion of particles is described by the well-established Newton's laws of motion. The governing equations for the translational and rotational motion of particle i with mass m_i and moment of inertia I_i can be written as:

$$m_i \frac{d\mathbf{v}_i}{dt} = \sum \left(\mathbf{F}_{ij}^n + \mathbf{F}_{ij}^s + m_i \mathbf{g} \right), \tag{1}$$

and

$$I_i \frac{d\boldsymbol{\omega}_i}{dt} = \sum \left(\mathbf{R}_i \times \mathbf{F}_{ij}^s - \mu_r R_i \left| \mathbf{F}_{ij}^n \right| \hat{\boldsymbol{\omega}}_i \right), \tag{2}$$

where $\mathbf{v}_i$, $\boldsymbol{\omega}_i$, and I_i are the translational velocity, angular velocity, and moment of inertia of the particle i, respectively, while $\hat{\boldsymbol{\omega}}_i$ represents a unit vector equal to $\boldsymbol{\omega}_i$ divided by its magnitude. $\mathbf{R}_i$ is a vector running from the centre of the particle to the contact point with its magnitude equal to particle radius R_i. $\mathbf{F}_{ij}^n$ and $\mathbf{F}_{ij}^s$ represent, respectively, the normal contact force and the tangential contact force imposed on particle i by particle j and $m_i\mathbf{g}$ is the gravitational force. The first part of the right hand side in Eq. (2) is the torque caused by tangential force $\mathbf{F}_{ij}^s$ and the second part is the rolling friction torque caused by normal force $\mathbf{F}_{ij}^n$, where μ_r is the coefficient of rolling friction.

Here we use the simplified Hertz–Mindlin and Deresiewicz model (Mindlin & Deresiewicz, 1953) where contact forces are given by,

$$F_{ij}^n = \left[\frac{2}{3} E \sqrt{\bar{R}} \xi_n{}^{\frac{3}{2}} - \gamma_n E \sqrt{\bar{R}} \sqrt{\xi_n} \left(v_{ij} \cdot \hat{\mathbf{n}}_{ij} \right) \right] \hat{\mathbf{n}}_{ij}, \tag{3}$$

and

$$F_{ij}^s = -sgn\left(\xi_s\right) \mu \left| \mathbf{F}_{ij}^n \right| \left[\frac{1 - \left(1 - \min\left(\xi_s, \xi_{s,\max}\right)\right)^{3/2}}{\xi_{s,\max}} \right] \hat{\boldsymbol{\xi}}_s, \tag{4}$$

where $E = Y/(1-\tilde{\sigma}^2)$, and Y and $\tilde{\sigma}$ are, respectively, Young's modulus and Poisson's ratio; ξ_n is the overlap between particles i and j; $\hat{\mathbf{n}}_{ij}$ is a unit vector running from the centre of particle j to the centre of particle i; $\bar{R} = R/2$ for mono-sized particle. The normal damping constant, γ_n, is the material property directly linked to the coefficient of restitution e. ξ_s and $\xi_{s,\max}$ are, the total and maximum tangential displacements of particles during contact respectively. $\hat{\boldsymbol{\xi}}_s$ is the unit vector of ξ_s. The results will be analysed in terms of impact energy which is given by NC_eC_f where N is the number of particles, C_e collision energy and C_f collision frequency. Collision energy is defined as kinetic energy which is given by $1/2 m_i v_{ij}^2$, where m_i is the mass of a particle and $v_{ij} \left(= \left| v_i - v_j \right| \right)$ is the relative collision velocity between two particles. Collision frequency is defined as the number of collisions per particle per second.

The required number of particles for each simulation was generated by varying size and distribution of the sample particles. Table 1 shows the material properties assigned to particles. Fig. 1 shows the autogeneous mill used in the simulations. A mill of 600 mm diameter and 320 mm length has been selected for the simulation. The mill has eight square-shaped lifters of 6 mm cross-section. The mill diameter was selected according to the thesis work by Banini

Table 1
Model parameters used in DEM.

Parameter	Base value	Varying range
Number of particles, N	14,500	–
Particle density, ρ (kg/m^3)	2.65×10^3	–
Young's modulus, Y (N/m^2)	1×10^6	1×10^7, 1×10^8 (μ=0.4)
Poisson's ratio, $\tilde{\sigma}$	0.29	
Particle/particle sliding friction coefficient, μ	0.4	0.1, 0.8 ($Y = 1 \times 10^7$)
Particle/mill sliding friction coefficient	0.3	–
Restitution coefficient, e	0.68	–
Rotation speed, ω (rpm)	40	27.3, 32.7, 38.2