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Evaluation of particle packing models by comparing with published test results

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ABSTRACT

The existing particle packing density models each with two or more parameters accounting for certain particle interactions (the loosening effect parameter, wall effect parameter, wedging effect parameter, and compaction index, denoted by *a*, *b*, *c*, and *K*, respectively) may be classified into the 2-parameter model (with *a* and *b* incorporated), the compressible model (with *a*, *b*, and *K* incorporated), and the 3-parameter model (with *a*, *b*, and *c* incorporated). This paper evaluates these models by comparing their respective packing density predictions with the test results published in the literature. It was found that their accuracy varies with both the size ratio and volumetric fractions of the binary mix. In general, when the size ratio is larger than 0.65, all the packing models are sufficiently accurate. However, when the size ratio is smaller than 0.65, some of them become inaccurate and the errors tend to be larger at around the volumetric fractions giving maximum packing density. Relatively, the 3-parameter model is the most accurate and widely applicable.

1. Introduction

Particle packing is central in many branches of materials science, powder technology, and processing industry because it governs the behavior of granular materials. There are several structural effects affecting the way solid particles of different sizes are packed together. These include the *filling effect* of the fine particles filling into the voids of the coarse particles, the *occupying effect* of the coarse particles occupying solid volumes in place of porous bulk volumes of the fine particles, the *loosening effect* of the fine particles loosening the packing of the coarse particle when squeezing themselves into the voids of the coarse particles, and the *wall effect* of the coarse particles disrupting the packing of the fine particles at the wall-like boundaries of the coarse particles (Alexander & Mindess, 2005; De Larrard, 1999; Kwan, Chan, & Wong, 2013). The filling and occupying effects would increase the packing density, while the loosening and wall effects would decrease the packing density. Therefore, understanding these structural effects is crucial in predicting the packing density of a particle system.

Whilst the filling and occupying effects are fundamental and straight forward, the loosening and wall effects are quite complicated and dependent on both the size ratio (ratio of the size of the fine particles to the size of the coarse particles) and the volumetric fractions (each volumetric fraction is the solid volume of one particular size class of particles, expressed as a fraction of the solid volume of all particles). Some earlier particle packing models do not separately account for the loosening and wall effects (Aïm & Goff, 1968; Dewar, 1999; Goltermann, Johansen, & Palbøl, 1997; Powers, 1968; Toufar, Klose, & Born, 1977). Later and more advanced particle packing models separately account for the loosening and wall effects in terms of two parameters called the loosening effect parameter and wall effect parameter (De Larrard, 1999; Stovall, de Larrard, & Buil, 1986; Yu & Standish, 1987, 1991; Yu, Zou, & Standish, 1996; Yu, Bridgwater, & Burbidge, 1997). These two parameters are expressed as interaction functions of the size ratio, obtained by regression analysis of the packing density test results of binary mixes of mono-sized particles with varying size ratio. With these two parameters and their respective interaction functions incorporated to account for the loosening and wall effects, such models may be collectively called the 2-parameter models.

For brevity, the loosening effect parameter is denoted by *a*, whereas the wall effect parameter is denoted by *b*. Throughout the years, the 2-parameter model has been refined to improve accuracy and applicability. For instance, Yu et al. (1996) refined the 2-parameter model by introducing the concept of equivalent packing diameter to allow for the influence of particle shape. Subsequently, Yu et al. (1997) further refined the 2-parameter model by taking into account also the agglomeration of very fine particles caused by cohesive inter-particle forces. Later, De Larrard (1999)

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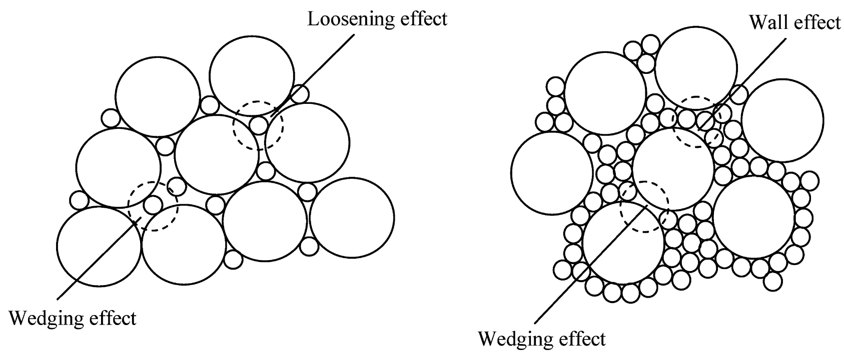

Fig. 1. A schematic diagram showing the loosening, wall, and wedging effects.

incorporated a compaction index (denoted by K) to allow for the effect of compaction during packing. To distinguish his model from the others, he named this model a compressible packing model. In fact, with the parameters a, b, and K incorporated, this model is no longer a 2-parameter model and should be classified into a distinctive class of its own.

The accuracy of different particle packing models has been evaluated and compared in two important studies. As reported by Johansen and Andersen (1989), Petersen has evaluated the accuracy of the models developed by Aïm and Goff (1968) and Toufar et al. (1977) by checking these models against the packing density test results of binary mixes of mono-sized particles obtained by Westman and Hugill (1930) and McGeary (1961). Petersen found that for binary mixes with size ratio smaller than 0.22, Aïm and Goff's model is more accurate whilst for binary mixes with size ratio larger than 0.22, Toufar et al.'s model is better. More recently, Jones, Zheng, and Newlands (2002) have evaluated and compared the accuracy of the models developed by Stovall et al. (1986), Goltermann et al. (1997), De Larrard (1999), and Dewar (1999) using the packing density test results obtained from their own experiments and those obtained by Standish and Borger (1979) and Goltermann et al. (1997). They pointed out that the accuracy of the four models differs from each other and varies with the size ratio, and gave a general conclusion that all these models seem to work better when the size ratio is larger than 0.40.

Recently, Kwan et al. (2013) identified a new particle interaction called the wedging effect. Like the loosening and wall effects, the wedging effect would also reduce the packing density. The wedging effect occurs when the fine particles are slightly less than or slightly more than enough to fill the voids between the coarse particles, as shown in Fig. 1. When the fine particles are slightly less than enough to fill the voids between the coarse particles, some isolated fine particles could be entrapped in the gaps between the coarse particles thereby wedging the coarse particles apart. When the fine particles are slightly more than enough to fill the voids between the coarse particles, some thin layers of fine particles at the gaps between the coarse particles could be incompletely filled causing voids to be formed there and the apparent wedging of the fine particles against the coarse particles. Kwan et al. introduced an additional wedging effect parameter (denoted by c) to allow for such wedging effect. With three parameters a, b, and c incorporated to account for the loosening effect, wall effect, and wedging effect, respectively, this particle packing model is classified as a 3-parameter packing model.

With further advancement and development of particle packing models since the previous evaluation by Jones et al. (2002), it should now be an opportune time to evaluate the packing models developed up to this stage to find out their relative accuracy and applicability, and of course also to gauge our progress in recent years. In this paper, the 2-parameter model (with a and b incorporated), the compressible model (with a, b, and K incorporated), and the 3-parameter model (with a, b, and c incorporated) are evaluated by comparing their respective packing density predictions with the packing density test results published by Westman and Hugill (1930), Ridgway and Tarbuck (1968), and Kwan et al. (2013). Only the packing density test results of binary mixes of mono-sized and spherical/rounded particles are used for the evaluation because the focus is not on the effect of particle shape. It will be seen that in general the packing models tend to have larger errors within certain ranges of size ratio and volumetric fractions. Nevertheless, the incorporation of the wedging effect parameter c would significantly reduce such errors.

2. Particle packing models evaluated

Three types of particle packing models are evaluated herein. They are the 2-parameter model developed by Stovall et al. (1986), Yu and Standish (1987, 1991), and Yu et al. (1996), the compressible model developed by De Larrard (1999), and the 3-parameter model developed by Kwan et al. (2013). Before evaluation, a brief outline of these packing models is given in the following. It is noteworthy that in all these packing models, the parameters a, b, and c are derived as empirical formulas in terms of the size ratio s by regression analysis of the available test results.

2.1. The 2-parameter model

Consider a binary mix of fine particles (designated as size class 1) and coarse particles (designated as size class 2). Let the packing densities of size class 1 and size class 2 be ϕ_1 and ϕ_2, respectively, and the volumetric fractions of size class 1 and size class 2 be r_1 and r_2, respectively (note that $r_1 + r_2 = 1$). There is an optimum value of r_1 (denoted by r_1^*) or an optimum value of r_2 (denoted by r_2^*) that yields the maximum packing density. If r_1 is lower than r_1^* (or r_2 is higher than r_2^*), then size class 2 is dominant because the amount of fine particles would then be less than enough to fill the voids between the coarse particles. If r_1 is higher than r_1^* (or r_2 is lower than r_2^*), then size class 1 is dominant because the amount of fine particles would then be more than enough to fill the voids between the coarse particles.

When size class 2 is dominant, the filling and loosening effects would occur. In such case, the packing density of the binary mix (denoted by ϕ_2^*) may be obtained from the following equation, where a is the loosening effect parameter:

$$\frac{1}{\phi_2^*} = \left(\frac{r_1}{\phi_1} + \frac{r_2}{\phi_2}\right) - (1-a)\frac{r_1}{\phi_1}. \tag{1}$$