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Experimental determination of Drucker-Prager yield criterion parameters for normal and high strength concretes under triaxial compression

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HIGHLIGHTS

- Triaxial compression tests were performed on NSC ad HSC for different lateral pressures.
- Mohr circles were drawn by experimental data.
- Cohesion and internal friction angles were obtained by using Mohr circles.
- Drucker-Prager yield criterion parameters were determined.
- Some equations were proposed to calculate the Drucker-Prager parameters by using concrete compressive strength.

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ABSTRACT

In this study, parameters which define Drucker-Prager yield criterion were investigated experimentally for both normal strength concrete (NSC) and high strength concrete (HSC) by triaxial compression tests. Total 16 concrete series with different concrete strength were produced. While 8 concrete series were produced as NSC ($20\ \text{MPa} < f_{cm} < 55\ \text{MPa}$), other 8 series were produced as HSC ($55\ \text{MPa} < f_{cm} < 85\ \text{MPa}$). Each series contains 16 specimens. Triaxial compression tests were performed on these concrete specimens. Concrete compressive strength of the specimens were measured under 4 different lateral compression level (0, 1, 2 and 4 MPa). After Mohr circles were drawn by using the data obtained from triaxial compression tests, the values of cohesion were determined between 5 MPa and 13 MPa for NSC and 13 MPa and 19 MPa for HSC. The values of internal friction angle were also obtained between 27° and 34° for NSC, 34° and 39° for HSC from the Mohr circles. The parameters defining the Drucker-Prager yield criterion were determined by using the values of cohesion and internal friction angle. Finally, after some equations were proposed to calculate the parameters by using concrete compressive strength, the validation of these equations were also proved in this study.

1. Introduction

Behaviors of structures or structural members mainly depend on their materials under different loading conditions. In structural analyses, utilization of accurate material models for a structure or a structural member is very important. Accurate material models must be used to obtain accurate behavior of structures or structural members.

Mainly, material models can be classified into two groups. First group models includes linear material models. Second group models are non-linear material models. Non-linear material models allows to obtain whole response of a structure or a structural member under different loading conditions. So, they are preferred to perform the more sensitive and the more accurate analyses.

Creation of a mathematical or an analytical model for nonlinear behavior of a material is more difficult than creation of a linear material model. During the creation of nonlinear material model, the most important and the most difficult stage is determination of yield point of a material under loading.

Indeed, obtaining of yield strength from stress-strain graphics is very difficult for most materials. However, many methods and theories such as Maximum Principal Stress Theory (Rankine 1850), Maximum Shear Stress Theory (Tresca yield criterion), Maximum

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Principal Strain Theory (St. Venant), Total Strain Energy Theory (Beltrami theory), Distortion Energy Theory (Von Mises yield criterion) Mohr-Coulomb yield criterion, Drucker-Prager yield criterion, Bresler-Pister yield criterion, Willam-Warnke yield criterion, Hill's quadratic yield criterion, generalized Hill yield criterion, Hosford yield criterion [1] etc. were used to determine the yield strength in technical literature.

2. Drucker-Prager yield criterion

Drucker-Prager model is an elastoplastic model used widely for soils, concrete, and polymers. Drucker-Prager model was created by smoothing of the surface defined in the Mohr-Coulomb yield criterion and by simply modifying of the Von Mises yield criterion [1,2]. Drucker-Prager criterion includes the hydrostatic pressure effect on the shearing resistance of a material [1–3]. Generally this model can be expressed by the Eq. (1).

$$f(I_1, J_2) = \alpha I_1 + \sqrt{J_2} - k = 0 \tag{1}$$

In this equation, α and k are defined as positive constants at each point of the materials [1]. They are called also as Drucker-Prager parameters. I_1 and J_2 are defined as first invariant of Cauchy stresses and the second invariant of the deviatoric part of the Cauchy stresses respectively [1–3] Drucker-Prager model is expressed also by Eq. (2) by using the principal stresses ($\sigma_1, \sigma_2, \sigma_3$);

$$\sqrt{\frac{1}{6}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]} = k + \alpha(\sigma_1+\sigma_2+\sigma_3) \tag{2}$$

For uniaxial tension and uniaxial compression, Eq. (2) can be written as Eqs. (3) and (4) respectively[1]:

$$\frac{1}{\sqrt{3}}\sigma_t = k + \alpha\sigma_t \tag{3}$$

$$\frac{1}{\sqrt{3}}\sigma_c = k - \alpha\sigma_c \tag{4}$$

In the Eqs. (3) and (4), σ_t and σ_c are yield strength under tension and compression. The Drucker-Prager parameters are written using two following equations Eqs. (3) and (4) [1–3].

$$\alpha = \frac{1}{\sqrt{3}}\left(\frac{\sigma_t - \sigma_c}{\sigma_c + \sigma_t}\right) \tag{5}$$

$$k = \frac{2}{\sqrt{3}}\left(\frac{\sigma_c \sigma_t}{\sigma_c + \sigma_t}\right) \tag{6}$$

Indeed, it is very difficult to calculate the Drucker-Prager parameters for concrete by using the Eqs. (5) and (6). Because it is almost impossible to determine the yield strengths (σ_t, σ_c) of concrete under tension and compression.

The formulas used in the calculation of the Drucker-Prager parameters can also be obtained by using the Mohr-Coulomb yield criterion. As mentioned above, Drucker-Prager yield surface is smoothing form of the Mohr-Coulomb yield surface [1]. Mohr Coulomb yield function is seen as a pyramid with irregular hexagonal base in three-dimensional principle stress space. [1]. Drucker-Prager yield function is seen as a cone in the same stress space (Fig. 1). The two functions produce same values at A and B points. If both of those functions matched for point A, Drucker-Prager Parameters can be calculated by Eqs. (7) and (8) [1].

$$\alpha = \frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)} \tag{7}$$

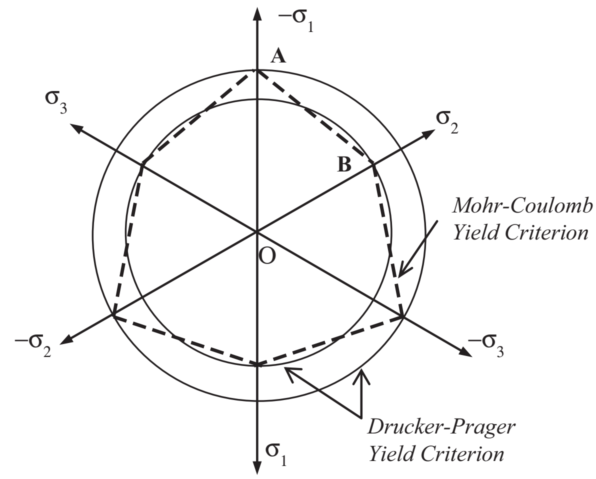

Fig. 1. Shapes of Mohr-Coulomb and Drucker-Prager yield criterions in principal coordinates [1].

$$k = \frac{6c\cos\phi}{\sqrt{3}(3-\sin\phi)} \tag{8}$$

For point B, corresponding parameters are calculated by Eqs. (9) and (10).

$$\alpha = \frac{2\sin\phi}{\sqrt{3}(3+\sin\phi)} \tag{9}$$

$$k = \frac{6c\cos\phi}{\sqrt{3}(3+\sin\phi)} \tag{10}$$

In these equations, c and ϕ are cohesion and internal friction angle respectively.

3. Literature overview

Drucker-Prager yield criterion has been successfully used in many nonlinear analyses performed on concrete for about last three decades. In these analyses, nonlinear behaviors of concretes or structural members were examined and/or verified by using D-P Criterion. This type studies are more common in literature. In the other type studies, c and ϕ parameters used in the constitution of D-P yield criterion were investigated for different concrete strengths, different concrete types (such as FRP Concrete, Normal Strength Concrete), different loading cases and etc. Some examples about these two type studies were presented below.

Doran et al. [4] used c = 2.8–3.4 MPa and ϕ = 25–38° in their study by referring to the studies done by Lubnier et al. [5] and Oller et al. [6]. They also proposed an equation (Eq. (11)) to calculate concrete cohesion. Calayır and Karaton [7] used Drucker-Prager material models with c = 2.109 MPa and ϕ = 38° values for fc′ = 25 MPa without referring to any study in the non-linear dynamic analysis of arch dams. By referring to the studies of Doran et al. [4] and Polat et al. [8], Arslan [9] used Eq. (11) in the calculation of cohesion of concrete (f_{ck} = 22.5 MPa).

$$c = 0.23\ln(E_o D_{max}^2) - 0.6 \tag{11}$$

In this equation, E_o and D_{max} are young modulus and maximum aggregate size respectively. Arslan [10] also investigated the sensitivity of the Drucker–Prager modeling parameters and the use of the model in plasticity theory for shear design. He stated that ϕ = 30–37° value range can be used generally for concrete. However he used ϕ = 37° value for high strength concrete and Eqs. (11) and (12) to calculate concrete cohesion.