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Characterization of asphalt concrete linear viscoelastic behavior utilizing Havriliak–Negami complex modulus model

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HIGHLIGHTS

- Both continuous relaxation and retardation spectra were analytically derived from a complex modulus model.
- A numerical interconversion method was proposed to determine one continuous spectrum from the other.
- High-quality discrete relaxation and retardation spectra were directly determined from the continuous ones.
- A two-step method for determining reduced master curves was suggested.

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ABSTRACT

This study proposed a new procedure for characterizing linear viscoelastic (LVE) behavior of asphalt concrete using the Havriliak–Negami (HN) complex modulus model and corresponding continuous relaxation spectrum model. To extend its application to modeling retardation behavior of asphalt concrete, the continuous retardation spectrum was analytically derived from the HN model by performing an inverse Fourier–Laplace transform followed by a variable substitution. The two continuous spectra of the HN model allow the accurate construction of all the modulus and compliance function master curves in time and frequency domains as well as the efficient determination of the high-quality discrete relaxation and retardation spectra. The advantages of the proposed approach over existing ones were demonstrated through two different complex modulus test data sets. Also, a simple and practical numerical interconversion method was presented based on the LVE relations between the continuous relaxation and retardation spectra, which can effectively compute one continuous spectrum from the other for any characterization model. Further, for practical considerations, a two-step method for determining reduced master curves with fewer discrete spectrum lines was suggested.

1. Introduction

It is well known that asphalt concrete behaves as a linear viscoelastic (LVE) solid under small strain levels (typically below 150 $\mu\varepsilon$) and a broad range of temperature and frequency. As such, the LVE properties of asphalt concrete can be characterized using the generalized Maxwell (GM) model and generalized Voigt (GV) model [1–3]. A principal advantage of the two mechanical models with different configurations of linear springs and dashpots is that they mathematically yield the discrete relaxation and retardation spectra, respectively. In terms of the discrete spectra, the time-domain modulus and compliance functions, commonly called as Prony Series representations, can be analytically converted into the frequency-domain ones and efficiently implemented into numerical computation of the hereditary integrals in the LVE constitutive equations [3–10].

Numerous approaches have been presented to determine the discrete spectra from experimental data. Schapery [11] reported the collocation method in which the spectrum strengths are solved by using only the data at the preselected collocation points. Cost and Becker [12] considered the whole set of laboratory data and developed the multiple data method through a least squares fitting technique conducted in Laplace-transform domain. These schemes have been widely used in practice; however, they may cause two common issues, i.e., spectrum oscillations and negative spectrum strengths, due to scatters in the data. Spectrum oscillations are one of the principal sources of the waviness in the master curves and negative spectrum strengths are physically uninterpretable.

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To address these problems, Parker and Kim [3] designed a power-law series pre-smoothing method that eliminates the scatters in the raw data. Emri and Tschoegl [13] and Tschoegl and Emri [14] proposed a recursive algorithm in light of the graphical characteristics of the exponential kernel functions. Others focused their research on resolving the ill-posed (or non-unique) problems during the identification of the discrete spectra [15–17]. Still others established different interconversion methods to indirectly calculate one discrete spectrum from the other [18–20].

In recent years, several attempts have been made to develop continuous spectrum methods. The continuous spectra can be treated as the limiting case of the discrete ones, in which the time constants are spaced infinitely closely. The use of the continuous spectra can effectively avoid the issues occurring in the determination of the discrete spectra, such as negative spectrum strengths, wavy master curves and excessive parameters. Levenberg and Shah [21] and Levenberg [22] used a lognormal distribution function to model the continuous relaxation spectrum. However, this function is known to be completely symmetrical on the logarithmic scale, and therefore is not applicable to all asphalt mixtures. Mun and Zi [23] applied an inverse Laplace transform to a relaxation modulus representation in time domain, obtaining the high order approximations of the continuous relaxation spectrum. Obviously, the two methods only allow for the characterization of the asphalt concrete relaxation behavior, not providing mathematical forms for the continuous retardation spectrum. Although, theoretically, the two continuous spectra can be interconverted into each other, such interconversion methods can hardly be found in the literature. Bhattacharjee et al. [24] extracted the continuous relaxation spectrum from the storage modulus represented by the sigmoidal model, and determined the corresponding continuous retardation spectrum by applying the same procedure to the storage compliance. Nevertheless, a major issue concerning this method is that the LVE relations between the continuous spectra cannot be guaranteed as discussed later.

To overcome the aforementioned problems, the present paper proposes a procedure for characterizing LVE behavior of asphalt concrete using a complex modulus model. Both continuous relaxation and retardation spectra can be analytically derived from this model. Also, a simple and practical numerical interconversion method was considered to determine one continuous spectrum from the other, and vice versa. Further, for practical considerations, a two-step method for determining reduced master curves was suggested.

2. Theoretical background

For LVE materials, the one-dimensional constitutive relationship between the stress σ and strain ε, under isothermal and non-aging conditions, can be expressed by the Boltzmann superposition integrals [5]

$$\sigma(t) = \int_{0^-}^{t} E(t-\zeta)\frac{d\varepsilon}{d\zeta}d\zeta \tag{1}$$

$$\varepsilon(t) = \int_{0^-}^{t} D(t-\zeta)\frac{d\sigma}{d\zeta}d\zeta \tag{2}$$

where t is the time of interest; $E(t)$ is the relaxation modulus; $D(t)$ is the creep compliance; and ζ is an integral variable. Eqs. (1) and (2) represent two equivalent forms, corresponding to an applied strain history and an applied stress history, respectively.

The relaxation modulus $E(t)$ of LVE solids is commonly represented by the GM model that is composed of a spring and a series of Maxwell elements in parallel. The analytical expression can be given by a Prony series [5]

$$E(t) = E_e + \sum_{i=1}^{N} E_i e^{-t/\tau_i} = E_g - \sum_{i=1}^{N} E_i(1-e^{-t/\tau_i}) \tag{3}$$

where τ_i and E_i are the ith relaxation time and relaxation strength, respectively; E_e is the equilibrium modulus, i.e., $\lim_{t\to\infty}E(t)$; and $E_g = E_e + \sum_{i=1}^{N}E_i$ is the glassy (or instantaneous) modulus, i.e., $\lim_{t\to 0}E(t)$. The finite number of parameters $\{\tau_i, E_i\}$ ($i = 1, \ldots, N$) in Eq. (3) constitute the so-called discrete relaxation spectrum, which represents a distribution of moduli over relaxation times. The accuracy of the Prony series representation in characterizing the LVE properties improves with the increasing number and density of the relaxation times. When the relaxation times are spaced infinitely closely, one can obtain a continuous relaxation spectrum $H(\tau)$. Accordingly, the relaxation modulus $E(t)$ is defined in an integral form as follows [5]

$$\begin{aligned} E(t) &= E_e + \int_{-\infty}^{\infty} H(\tau)e^{-t/\tau}d\ln\tau \\ &= E_g - \int_{-\infty}^{\infty} H(\tau)(1-e^{-t/\tau})d\ln\tau \end{aligned} \tag{4}$$

In terms of Carson transform, i.e., s-multiplied Laplace transform, $E(t)$ can be converted into the operational modulus $\tilde{E}(s)$ in Laplace-transform domain as shown below

$$\begin{aligned} \tilde{E}(s) &= s\int_{0}^{\infty} E(t)e^{-st}dt = E_e + \int_{-\infty}^{\infty} H(\tau)\frac{s\tau}{1+s\tau}d\ln\tau \\ &= E_g - \int_{-\infty}^{\infty} H(\tau)\frac{1}{1+s\tau}d\ln\tau \end{aligned} \tag{5}$$

where s is the Laplace transform variable. The complex modulus $E^*(\omega)$, storage modulus $E'(\omega)$ and loss modulus $E''(\omega)$ in frequency domain can be determined by replacing s with $i\omega$, as displayed in Eqs. (6)–(8)

$$E^*(\omega) = \tilde{E}(s)|_{s=i\omega} = E'(\omega) + iE''(\omega) \tag{6}$$

$$\begin{aligned} E'(\omega) &= E_e + \int_{-\infty}^{\infty} H(\tau)\frac{\omega^2\tau^2}{1+\omega^2\tau^2}d\ln\tau \\ &= E_g - \int_{-\infty}^{\infty} H(\tau)\frac{1}{1+\omega^2\tau^2}d\ln\tau \end{aligned} \tag{7}$$

$$E''(\omega) = \int_{-\infty}^{\infty} H(\tau)\frac{\omega\tau}{1+\omega^2\tau^2}d\ln\tau \tag{8}$$

where $\omega = 2\pi f$ is the angular frequency; f is the loading frequency; and $i = \sqrt{-1}$ is the imaginary unit.

The GV model that consists of a spring and a group of Voigt elements in series is very convenient to model the creep compliance $D(t)$ of LVE solids. Likewise, the Prony series representation is given by [5]

$$D(t) = D_e - \sum_{i=1}^{N} D_i e^{-t/\tau_i} = D_g + \sum_{i=1}^{N}(1-D_i e^{-t/\tau_i}) \tag{9}$$

where τ_i and D_i are the ith retardation time and retardation strength, respectively; $D_e = 1/E_e$ is the equilibrium compliance, i.e., $\lim_{t\to\infty}D(t)$; and $D_g = D_e - \sum_{i=1}^{N}D_i = 1/E_g$ is the glassy (or instantaneous) compliance, i.e., $\lim_{t\to 0}D(t)$. The parameter set $\{\tau_i, D_i\}$ ($i = 1, \ldots, N$) is commonly termed as the discrete retardation spectrum, describing a distribution of compliance over retardation times. Note that often, the retardation time and relaxation time are denoted by the same symbol τ; however, they can be easily distinguished according to their viscoelastic functions. Similar to Eqs. (4)–(8), the compliance functions with respect to the continuous retardation spectrum $L(\tau)$ are formulated as follows [5]