



# Computational analysis of the creep behaviour of bituminous mixtures



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## HIGHLIGHTS

- A 3D constitutive model for the bituminous mixtures creep analysis was developed.
- The 3D model was characterised by inviscid plasticity and isotropic hardening.
- The Helmholtz free energy and the Clausius–Duhem inequality were introduced.
- The model was calibrated on the basis of experimental uniaxial creep recovery tests.
- The validation of the model was based on 3D FEM analysis of creep recovery tests.

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## ABSTRACT

This report introduces a visco-elasto-plastic constitutive model for the characterisation of stress–strain behaviour in bituminous mixtures. With the introduction of the Helmholtz free energy and using the concept of internal variables, it has been possible to express a plastic flow law, characterised by isotropic hardening. The formulation of the constitutive relationship has been developed in such a way as to verify a priori the universal dissipation principle, expressed by the Clausius–Duhem dissipative inequality. The model has been calibrated and validated on the basis of the creep recovery response of two different bituminous mixtures under various stress levels and loading time.

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## 1. Introduction

The definition of a constitutive model, which can grasp the true complexity of the response of bituminous mixtures to mechanical stresses, is an indispensable prerequisite for a rational analysis of the creep behaviour of asphalt concretes for road pavements, primarily with reference to the phenomenon of permanent deformation accumulation at high temperature. The visco-elasto-plastic response of bituminous mixtures can be described by means of rheological models comprising elastic, viscous and plastic components, paired in a more or less complex way, according to the accuracy desired for the simulation and the possibility of surmounting the mathematical complexity resulting from the number of initial components utilised [1–4]. A step forward in the constitutive modelling of asphalt concretes can be achieved, in terms of theoretical

generalisation of the rheological models, by means of the continuum thermomechanics, introducing concepts as the Helmholtz free energy and the dissipative inequality of Clausius–Duhem [5–8]. Alternative approaches rely on the fractional models [9–12], the continuum damage mechanics [13,14] or the distinct element method [15–18]. This paper, after providing a brief summary of a thermodynamically congruent visco-elastic model, presents and discusses a plastic flow law that completes the constitutive modelling framework, on the basis of inviscid plasticity and isotropic hardening. The study is integrated by the calibration and validation of the proposed model, on the basis of the creep recovery response of a Stone Mastic Asphalt and a Wearing Course Asphalt concrete.

## 2. Theory and calculation

### 2.1. Formulation of the visco-elastic constitutive model

The mechanical behaviour of a visco-elastic “material point” can be defined calculating a correlation for the Helmholtz free energy ( $\psi$ ) in comparison with an appropriate set of internal

Abbreviations: SMA, Stone Mastic Asphalt; WCA, Wearing Course Asphalt; FEM, finite element method.

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variables  $(\mathbf{q}_i)$ , which can describe micro-structural state of the point and associate each of these to a viscous process [19]:

$$\psi = \psi(\boldsymbol{\varepsilon}, \mathbf{q}_i) \quad i \in \{1, 2, \dots, n\} \quad (1)$$

This can be expressed as:

$$\psi(\boldsymbol{\varepsilon}, \mathbf{q}_i) = W^0(\boldsymbol{\varepsilon}) - \sum_{i=1}^n \mathbf{q}_i : \boldsymbol{\varepsilon} \quad i \in \{1, 2, \dots, n\} \quad (2)$$

where  $W^0(\boldsymbol{\varepsilon}) = \frac{1}{2} (\boldsymbol{\varepsilon} : \mathbf{D}_0 : \boldsymbol{\varepsilon})$  represents the deformation energy associated to the instantaneous elastic response in the hypothesis of linear elastic behaviour ( $\mathbf{D}_0$  represents the tensor of the instantaneous elastic modulus whereas  $\boldsymbol{\varepsilon}$  is the strain tensor).

For the principle of universal dissipation, the constitutive relation introduced must satisfy the dissipative inequality of Clausius–Duhem, which, in pure mechanics, can be formulated as:

$$\dot{\psi} - \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} \leq 0 \quad (3)$$

from which it follows that:

$$\left[ \frac{\partial W^0}{\partial \boldsymbol{\varepsilon}} - \sum_{i=1}^n \mathbf{q}_i : \boldsymbol{\sigma} \right] : \dot{\boldsymbol{\varepsilon}} - \sum_{i=1}^n \dot{\mathbf{q}}_i : \boldsymbol{\varepsilon} \leq 0 \quad (4)$$

This relation must be satisfied whatever the kinetic process  $\boldsymbol{\varepsilon}, \mathbf{q}_i$  and whatever the kinetic process action  $\dot{\boldsymbol{\varepsilon}}, \dot{\mathbf{q}}_i$ .

The following expressions for the Cauchy stress tensor and the terms of dissipation are thus obtained:

$$\boldsymbol{\sigma} = \frac{\partial W^0}{\partial \boldsymbol{\varepsilon}} - \sum_{i=1}^n \mathbf{q}_i = \boldsymbol{\sigma}^0 - \sum_{i=1}^n \mathbf{q}_i \quad (5)$$

$$\dot{\mathbf{q}}_i : \boldsymbol{\varepsilon} \geq 0 \quad \forall i \in \{1, 2, \dots, n\} \quad (6)$$

where  $\boldsymbol{\sigma}^0$  represents the instantaneous elastic response. It is possible to associate a relative stiffness  $\gamma_i$  and a relaxation time  $\tau_i$  to each viscous process. A stiffness is then introduced to the equilibrium  $\gamma_\infty$  so as to give:

$$\gamma_\infty + \sum_{i=1}^n \gamma_i = 1 \quad (7)$$

The evolution of the internal parameters can be defined using the following system of ordinary differential equations:

$$\dot{\mathbf{q}}_i + \frac{1}{\tau_i} \mathbf{q}_i = 2 \frac{\gamma_i}{\tau_i} \frac{\partial W^0}{\partial \boldsymbol{\varepsilon}} \quad (8)$$

to which the initial values are coupled:

$$\lim_{t \rightarrow -\infty} \mathbf{q}_i = 0 \quad (9)$$

The Cauchy problem allows a solution in closed form:

$$\mathbf{q}_i(t) = \frac{\gamma_i}{\tau_i} \int_{-\infty}^t \exp \left[ -\frac{(t-s)}{\tau_i} \right] \boldsymbol{\sigma}^0(s) ds \quad (10)$$

from which it follows that:

$$\boldsymbol{\sigma}(t) = \boldsymbol{\sigma}^0(t) - \sum_{i=1}^n \frac{\gamma_i}{\tau_i} \int_{-\infty}^t \exp \left[ -\frac{(t-s)}{\tau_i} \right] \boldsymbol{\sigma}^0(s) ds \quad (11)$$

The previous formulation gives the general framework of a visco-elastic model [19], developed on the basis of an energetic approach, which however cannot take into account the permanent deformation experimentally observed under creep loading of asphalt materials.

## 2.2. Introduction of plastic behaviour

In order to take into account any permanent plastic strain  $\boldsymbol{\varepsilon}^p$  in the asphalt concrete as a consequence of the loading history, the

function of density of Helmholtz free energy can be reformulated as follows:

$$\psi(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^p, \mathbf{q}_i) = W^0(\boldsymbol{\varepsilon}^{ve}) - \sum_{i=1}^n \mathbf{q}_i : \boldsymbol{\varepsilon}^{ve} \quad (12)$$

where  $\boldsymbol{\varepsilon}^{ve} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p$  represents the reversible visco-elastic component of the strain. Applying the fulfilment of the dissipative inequality of Clausius–Duhem, the stress–strain equation and thermodynamic restrictions on the model are obtained:

$$\boldsymbol{\sigma}(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^p, \mathbf{q}_i) = \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}} = \frac{\partial W^0}{\partial \boldsymbol{\varepsilon}} - \sum_{i=1}^n \mathbf{q}_i \quad (13)$$

$$(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) : \dot{\mathbf{q}}_i \geq 0 \quad (14)$$

Taking into account the formulation of the density function of elastic energy  $W^0(\boldsymbol{\varepsilon}^{ve}) = \frac{1}{2} (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) : \mathbf{D}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p)$  and the law of evolution of the internal viscous parameters, the following is arrived at:

$$\boldsymbol{\sigma}(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^p, \mathbf{q}_i; t) = \mathbf{D}_0(\boldsymbol{\varepsilon}(t) - \boldsymbol{\varepsilon}^p(t)) - \sum_{i=1}^n \frac{\gamma_i}{\tau_i} \int_{-\infty}^t \exp \left[ -\frac{(t-s)}{\tau_i} \right] \boldsymbol{\sigma}^0(s) ds \quad (15)$$

$$\boldsymbol{\sigma}^0(t) = \mathbf{D}_0(\boldsymbol{\varepsilon}(t) - \boldsymbol{\varepsilon}^p(t)) \quad (16)$$

The definition of a visco-elasto-plastic model requires the introduction of a yield criterion, as well as a law of evolution for the plastic strains. It has been verified that the asphalt concrete response is different under compressive or tensile stresses and moreover it is characterised by both volumetric and deviatoric strain [5]; however, in order to investigate the creep behaviour under monotonic compression load, the Von Mises yield criterion coupled to an isotropic hardening, can be reasonably assumed [7]. Therefore, the following yield criterion has been introduced:

$$\phi(\boldsymbol{\sigma}, \kappa) = F(\boldsymbol{\sigma}) - \sigma_y(\kappa) = 0 \quad (17)$$

where  $\phi(\boldsymbol{\sigma}, \kappa)$  is a plastic potential,  $F(\boldsymbol{\sigma}) = \sqrt{3J_2}$ , where  $J_2$  is the second invariant of the deviatoric stress tensor,  $\sigma_y(\kappa)$  is a function describing the limit value that  $F(\boldsymbol{\sigma})$  can take without any further plastic creep occurring,  $\kappa$  is a scalar valued hardening parameter.

In order to obtain a model of the “strain-driven” type, and therefore able to take into account the dependence of the yield conditions on the rate of deformation and the plastic flows caused by creep, it is appropriate that the function  $F(\boldsymbol{\sigma})$  does not depend on the effective stress  $\sigma$ , but rather on the instantaneous elastic stress  $\boldsymbol{\sigma}^0(t) = \mathbf{D}_0(\boldsymbol{\varepsilon}(t) - \boldsymbol{\varepsilon}^p(t))$ . A proper definition of the inelastic response of a visco-elasto-plastic material requires a further step besides the identification of a yield condition: a flow rule for the plastic strains has to be introduced. In the mathematical theory of plasticity [19], the evolution law for the plastic strains can be written in the form:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda} \mathbf{m} \quad (18)$$

where  $\dot{\lambda}$  is the plastic multiplier and  $\mathbf{m}$  describes the direction of the plastic flow. In computational plasticity, the Kuhn–Tucker loading–unloading conditions, as well as the consistency condition [19], are introduced, in order to stabilise the conditions so that plastic flow occurs:

$$\dot{\lambda} \geq 0, \quad \phi \leq 0, \quad \phi \dot{\lambda} = 0 \quad (19)$$

$$\dot{\phi} \dot{\lambda} = 0 \quad (20)$$

It follows that:

$$\partial \phi / \partial \boldsymbol{\sigma}^0 : \dot{\boldsymbol{\sigma}}^0 + \partial \phi / \partial \kappa \dot{\kappa} = 0 \quad (21)$$

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