Particuology 10 (2012) 710–721

Contents lists available at SciVerse ScienceDirect

Particuology

journal homepage: www.elsevier.com/locate/partic

Calculation of relative permeability in reservoir engineering using an interacting triangular tube bundle model

Jinxun Wang[a], Mingzhe Dong[a,b,*], Jun Yao[b]

[a] *Department of Chemical and Petroleum Engineering, University of Calgary, Calgary, AB, Canada T2N 1N4*
[b] *College of Petroleum Engineering, China University of Petroleum (East), Qingdao, Shandong, China*

ARTICLE INFO

Article history:
Received 19 October 2011
Received in revised form 22 March 2012
Accepted 10 May 2012

Keywords:
Interacting tube bundle model
Porous media
Modeling
Relative permeability
Imbibition

ABSTRACT

Analytical expressions of relative permeability are derived for an interacting cylindrical tube bundle model. Equations for determining relative permeability curves from both the interacting uniform and interacting serial types of triangular tube bundle models are presented. Model parameters affecting the trend of relative permeability curves are discussed. Interacting triangular tube bundle models are used to history-match laboratory displacement experiments to determine the relative permeability curves of actual core samples. By adjusting model parameters to match the history of oil production and pressure drop, the estimated relative permeability curves provide a connection between the macroscopic flow behavior and the pore-scale characteristics of core samples.

1. Introduction

Relative permeability curves are among the most important parameters for reservoir engineering calculations. From laboratory unsteady-state displacement experiments, relative permeability curves are commonly determined by the history matching method. A given set of relative permeability functions with adjustable parameters, such as the power law (Corey type) model, are usually assumed at first. By matching the history of fluid production and pressure drop, relative permeability curves for the studied fluid-rock system are determined. Although a good history match can be achieved, the relative permeability functions obtained in this manner lack the pore-level physical analysis. The pore structure, and the actual flow mechanisms of fluids in pore space, are not represented in the empirical functions. This paper presents the method of using interacting tube bundle models as a history matching tool for determining relative permeability curves. By adjusting the model parameters to match production history, the determined relative permeability curves provide a connection between the macroscopic flow behavior and the pore scale characteristics.

Pore-level modeling of multiphase flow in porous media is an important numerical approach to connect microscopic flow mechanisms with their macroscopic manifestations. The simplest type of pore-level models is the conventional tube bundle model, which has been studied extensively by several researchers (Payatakes, Tien, & Turian, 1973; Purcell, 1949; Scheidegger, 1953). The conventional model is composed of parallel cylindrical tubes with identical or varied tube radii. Based on this simple model, capillary pressure and relative permeability were expressed as functions of the pore (tube) size distribution of the studied porous media. In all of the above tube bundle models, there is no interaction between the fluids flowing in neighboring tubes.

Pore-scale network models in which interaction between fluids in adjacent capillaries is allowed were first introduced by Fatt (1956). Various forms of network model have been proposed and widely used in the studies of different displacement processes (Blunt & King, 1991; Jerauld & Salter, 1990; Larson, Scriven, & Davis, 1981; Lenormand, Touboul, & Zarcone, 1988; Øren, Bakke, & Arntzen, 1998). Dullien (1992) and Blunt, Jackson, Piri, and Valvatne (2002) published literature reviews of this extensive field of study. The main computation work is to solve the pressure field in the network model at each time step. Poiseuille's law relates pressure drop to flow rate. Conservation of volume and the assumption of incompressibility of fluids give rise to a system of equations for determining the pressure in each pore-body. The number of unknowns to be solved is the same as the number of pore-bodies, and the system of pressure equations is nonlinear due to the presence of capillary pressure. Over the years, network models have

* Corresponding author at: Department of Chemical and Petroleum Engineering, University of Calgary, Calgary, AB, Canada T2N 1N4. Tel.: +1 403 210 7642; fax: +1 403 284 4852.
E-mail address: mingzhe.dong@ucalgary.ca (M. Dong).

1674-2001/$ – see front matter
http://dx.doi.org/10.1016/j.partic.2012.05.003

Nomenclature

A	cross sectional area of triangle (m^2)
A_c	cross sectional area of a porous medium (m^2)
A_o	area occupied by oil (m^2)
A_w	area occupied by water (m^2)
A_{wc}	corner area occupied by water in triangular tube (m^2)
G	shape factor of triangle
K_a	absolute permeability (m^2)
K_o	oil phase effective permeability (m^2)
K_{ro}	oil phase relative permeability
K_{rw}	water phase relative permeability
K_w	water phase effective permeability (m^2)
L	model length (m)
m	total number of tubes in which oil was filled
n	total tube number
ΔP	pressure drop (Pa)
Per	perimeter of triangle (m)
Q	fluid flow rate (m^3/s)
r	radius of curvature of the oil/water meniscus in the edges (m)
r_d	arc meniscus curvature radius when stable front interface advances in a triangular tube during drainage process (m)
r_{eff}	effective radius occupied by oil in a triangular tube (m)
r_i	radius of i-th tube (m)
r_{inc}	inscribed radius of triangular tube (m)
r_{max}	maximum inscribed radius of triangular tube (m)
r_{min}	minimum inscribed radius of triangular tube (m)
$\bar{r}$	average inscribed radius of triangular tube (m)
S_{or}	residual oil saturation
S_w	water saturation
$S_w{}^*$	normalized water saturation
S_{wi}	initial water saturation
Δx	interval length between two neighboring interfaces along the model (m)

Greek letters

η	adjustment parameter for effective radius of oil flow
ε	adjustment parameter for tortuosity
β	dimensionless resistance factor
μ	fluid viscosity (Pa s)
τ	tortuosity

Subscripts

i, j, k, l	tube size sequence number with 1 representing the largest tube size
o	oil phase
w	water phase

become increasingly sophisticated and, as a result, called for great demands on computational resources. Thus, this requirement of extensive computation inevitably places a restriction on the size of network models. Generally, the pore-scale network model represents only a sample of approximately 1 mm to a few centimeters cube (Blunt et al., 2002).

The simplicity of the conventional tube bundle models still attracts the attention of many researchers. Bartley and Ruth (1999) and Bartley and Ruth (2001) analyzed relative permeabilities using a bundle of serial tubes. Hui and Blint (2000) studied two-phase and three-phase relative permeabilities using a bundle of tubes of different sizes and with constant triangular cross-sections. Dahle, Celia, and Hassanizadeh (2005) investigated the dynamic effects of the capillary pressure–saturation relationship using a simple tube bundle model. To overcome the main drawback of deficient interconnectedness, Dong, Dullien, and Zhou (1998) and Dong, Dullien, Dai, and Li (2005, 2006) proposed an interacting tube bundle model in which pressure equilibration was imposed at any location along the model. The physical basis of this model is the observation that in one-dimensional immiscible displacement, at any cross section perpendicular to the direction of flow, the pressure in each phase is the same over the entire cross-section. The pressure difference between the two phases is the corresponding capillary pressure at that location. The flow characteristics of immiscible displacement in porous media have been successfully evaluated using the interacting tube bundle model. The interacting uniform type model was extended to an interacting serial type triangular tube bundle model by Wang and Dong (2011). To characterize the tortuous fluid flow in actual porous media, the interacting serial type model was constructed by connecting a series of interacting uniform type models in sequence. The same tube size distribution was used for all the segments of the model. In other words, along the model length, from segment to segment, the size of each individual tube varies, while the tube size distribution is the same at any location. The major advantage of the interacting tube bundle model over the sophisticated pore-scale network model is the simplicity in construction and computation.

In this paper, relative permeability curves are determined from interacting tube bundle models, starting from the cylindrical tube bundle model, to the interacting uniform type triangular tube bundle model, and then to the interacting serial type model. The interacting tube bundle model is then applied as a history matching tool to determine relative permeability curves from displacement experiments.

2. Relative permeability curves from interacting tube bundle models

Relative permeability is a measure of the flow ability of one fluid in a porous medium in the presence of other fluids, relative to the flow ability of the same fluid under a single-phase flow condition. Two-phase relative permeability curves are graphically represented as plots of relative permeabilities versus the wetting phase saturation. By calculating wetting phase occupancy and the flow abilities of wetting and nonwetting phases at different locations in an interacting tube bundle model, relative permeability curves can be obtained.

2.1. Interacting circular tube bundle model

The interacting capillary bundle model proposed by Dong et al. (1998, 2005, 2006) provides an idealized model for characterizing multiphase flow in porous media. The concept of pressure equilibration introduced in their model makes it possible to have analytical expressions for relative permeabilities. Fig. 1 shows an imbibition process in an interacting tube bundle model consisting of circular tubes with radii r_i (i = 1, 2, ..., n). In this figure (and henceforth in this paper), water is the wetting phase in the model, while oil is the non-wetting phase. The relative permeability expressions for the interacting tube bundle model are derived as follows.

The length of the interval between the interfaces in tubes k and $k+1$, as shown in Fig. 1, is Δx, and the pressure drops in water and oil phases are ΔP_w and ΔP_o, respectively. Tubes 1 to k are fully filled with oil over this interval, while tubes $k+1$ to n are fully filled with