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Analysis of shock wave reflection from fixed and moving boundaries using a stabilized particle method

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ABSTRACT

In the present paper, the efficiency of an enhanced formulation of the stabilized corrective smoothed particle method (CSPM) for simulation of shock wave propagation and reflection from fixed and moving solid boundaries in compressible fluids is investigated. The Lagrangian nature and its accuracy for imposing the boundary conditions are the two main reasons for adoption of CSPM. The governing equations are further modified for imposition of moving solid boundary conditions. In addition to the traditional artificial viscosity, which can remove numerically induced abnormal jumps in the field values, a velocity field smoothing technique is introduced as an efficient method for stabilizing the solution. The method has been implemented for one- and two-dimensional shock wave propagation and reflection from fixed and moving boundaries and the results have been compared with other available solutions. The method has also been adopted for simulation of shock wave propagation and reflection from infinite and finite solid boundaries.

1. Introduction

Shock waves are encountered in many present day industrial and engineering applications that experience various kinds of impacts or are subject to extreme conditions such as blast. Existence of a discontinuity or rapid changes and high gradients of field variables, have long been a major challenge for continuum based computational methods for solving fluid dynamics problems. The problem is further complicated if the model is expected to simulate interaction of a shock wave and a moving finite solid boundary.

Analysis of fluids dynamic problems are traditionally performed by Eulerian based methods, such as the finite difference method (FDM) and the finite volume method (FVM) (Liu & Liu, 2003). Eulerian based methods are often preferred for steady state problems, where no moving boundaries or material interfaces or evolving surfaces are encountered. Despite enormous efforts to resolve the mentioned difficulties in transient problems, it still remains an active computational challenge.

In a Lagrangian description, however, material and field variables are attached to freely moving nodes. As a result, enforcing moving boundaries and material interfaces are readily applied by defining the boundary values on boundary nodes. In addition, tracing of an evolving surface becomes rather simple due to possibility of tracing the particles on the initial free surface. The finite element method (FEM), as the most powerful Lagrangian based method, provides tremendous capabilities for solution of various solid, fluid and fluid–solid interaction problems. Nevertheless, its Lagrangian nature may cause mesh distortions and a major loss of accuracy. Available remedies include adaptive finite element analysis (Zienkiewicz, Taylor, & Nithiarasu, 2005), which necessitates remeshing algorithms and time consuming data transfer procedures, and the arbitrary Lagrangian Eulerian (ALE) method. Although ALE has been designed to benefit from both Eulerian and Lagrangian descriptions, it may encounter considerable numerical errors for large deformation problems undergoing high gradient fields (Li & Liu, 2002).

On the other hand, rapid development of meshless particle methods has now well extended to analysis of fluid flow and fluid–solid interaction problems. Among them, the Lagrangian based smoothed particle hydrodynamics (SPH), introduced in the 70s by Gingold and Monaghan (1977) and Lucy (1977), avoids any mesh distortion by virtue and is capable of enforcing various moving boundaries and interfaces much simpler than Eulerian based methods.

Several authors have simulated unsteady gas-dynamics shock wave problems. The first attempt can be traced back to Monaghan and Gingold in 1983. McCormick and Miller (1994, 1996) simulated one-dimensional shock tubes and studied propagation and

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reflection of shock waves by adopting simple artificial viscosity and repulsive force techniques to stabilize the particle method. Swegle and Attaway (1995) investigated the feasibility of using smoothed particle hydrodynamics for underwater explosion calculations.

Additionally, Brookshaw (2003) and Omang, Børve, and Trulsen (2003) extended the method of SPH to axisymmetric cylindrical coordinates in order to solve two-dimensional shock problems, while Liu, Liu, and Lam (2003b) proposed a new quartic smoothing function to analyse one-dimensional shock and TNT detonation problems. Liu, Liu, Lam, and Zong (2003) also adopted SPH to simulate the problem of water mitigation as a measure of reducing blast effects. Two different types of virtual particles were proposed to simulate the fluid/solid boundary conditions, and both contact and non-contact water mitigation simulations were performed.

Despite a number of advantages, SPH has encountered several drawbacks. It lacks the consistency condition in its discretized form, and cannot accurately approximate the solution on and near boundaries. Also it suffers various instabilities, such as the tensile instability, zero energy modes and high frequency oscillations. There have been a number of modifications to stabilize SPH (Li & Liu, 2002). The most popular stabilization procedure has been the method of artificial viscosity, which prevents the inter-particle penetration by inducing an artificial damping. One of the frequently used forms was introduced by Monaghan and Gingold (1983) and Gray, Monaghan, and Swift (2001). A corrective smoothed particle method CSPM was proposed by Chen et al. (Chen & Beraun, 2000; Chen, Beraun, & Carney, 1999; Chen, Beraun, & Jih, 1999b; Chen, Beraun, & Jih, 1999c; Chen, Beraun, & Jih, 2001) to address the tensile instability and boundary deficiency problems that have hampered full exploitation of standard SPH.

Recently, Ostad-Hossein and Mohammadi (2008, 2009) proposed a field smoothing stabilization approach for solid mechanics and hydrodynamics which avoids the necessity of stabilizing terms or definition of stress points. In this paper, this simple and efficient approach is further extended for implementation in shock propagation and reflection in compressible fluids. The field smoothing stabilization can remove high frequency extra vibrations which pollute the solution due to the truncation of Taylor series expansion by applying a second independent CSPM smoothing approximation of the target field value from the field values of its surrounding particles (Ostad-Hossein & Mohammadi, 2008).

Other improvements of the original CSPM method were reported by Liu, Liu, and Lam (2003a) to include the presence of discontinuity in simulating 1D shock waves by adopting a piecewise continuous formulation for each side of a discontinuity and adding a corrective part to the primary formulation of CSPM.

This paper is dedicated to the analysis of shock waves from different boundary conditions by a new stabilized CSPM methodology. Several numerical tests are discussed to assess the performance and accuracy of the proposed stabilizing approach. First, one-dimensional shock tube problems with different boundary conditions are comprehensively discussed and potential advantages and drawbacks are addressed. Available analytical solutions and other reference numerical results are used to compare and assess the obtained results. The final set of tests are related to the propagation, reflection and dispersion of two-dimensional shock waves from different boundary conditions in the form of infinite and finite solid walls. The discussion will be closed by providing the concluding remarks.

2. Gas dynamics

The governing equations in fluid dynamics including conservation of mass, momentum and energy can be written as a set of partial differential equations in a Lagrangian description. In the case of an inviscid fluid, the viscosity terms are discarded and the basic equations of conservation of mass, momentum and energy and the equation of state can be re-written as

$$\frac{D\rho}{Dt} = -\rho\vec{\nabla}\cdot\mathbf{v} \tag{1}$$

$$\frac{D\vec{\mathbf{v}}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p \tag{2}$$

$$\frac{Du}{Dt} = -\frac{p}{\rho}\vec{\nabla}\cdot\mathbf{v} \tag{3}$$

$$p = p(\rho, u) \tag{4}$$

For an ideal gas, the following equation of state is adopted

$$p = (\gamma-1)\rho u \tag{5}$$

where γ has the value of 1.4 for diatomic gases.

A wave reflection is anticipated when a shock wave, passing through a soft medium, reaches a denser medium or a solid surface/object. Depending on the obstacle size and geometry, a particular dispersion phenomenon may also occur. The reflection will be accompanied with an increase in density and pressure of the domain because of the combination of the incident and reflected waves. Rankine and Hugoniot (1851–1887) were the first to investigate the shock propagation and normal reflection overpressure in an ideal gas based on the conservation laws. According to their studies, the velocity of gas particles behind the shock front, u_s, can be defined as (Hugoniot, 1887, 1889; Rankine, 1870):

$$u_s = \frac{c_0}{\gamma}\left(\frac{p_1-p_0}{p_0}\right)\left[1+\left(\frac{\gamma+1}{2\gamma}\right)\left(\frac{p_1-p_0}{p_0}\right)\right]^{-(1/2)} \tag{6}$$

where c_0 is the sound velocity in the ambient air, p_0 is the ambient air pressure and p_1 is the pressure behind the shock front.

The reflected pressure from a fixed solid boundary, p_2, can also be calculated from:

$$\frac{p_2}{p_1} = \frac{((3\gamma-1)/(\gamma+1))(p_1/p_0)-((\gamma-1)/(\gamma+1))}{1+((\gamma-1)/(\gamma+1))(p_1/p_0)} \tag{7}$$

Eqs. (6) and (7) will be used for verification of results of shock wave propagation and reflection obtained from the proposed stabilized particle method.

3. Enforcing boundary conditions

From the physical point of view, there are generally two types of fluid behavior on solid boundaries. The first type is the non-slip boundary for viscous fluids. In this assumption, the relative velocity of fluid and boundary vanishes adjacent to the solid boundary. Another type is the slip boundary in inviscid fluids for which only the normal component of the relative velocity is assumed to be zero.

The presence of discontinuity due to solid boundary effects requires a modification of the velocity divergence terms in the mass continuity and energy conservation Eqs. (1) and (3).

A general reference volume of a one-dimensional fluid flow is considered at the solid boundary position. The particles can enter the volume with a velocity of v_x and leave it with a boundary affected velocity, as depicted in Fig. 1. The relative velocity of fluid flow and boundary surface is assumed to be zero, resulting in

$$v_x + \frac{\partial v_x}{\partial x}dx = v_{\mathrm{BND}} \tag{8}$$

where dx is the length of the reference volume in which particle impacts have occurred and v_{BND} is the velocity of boundary surface.