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Simulations of granular flow along an inclined plane using the Savage–Hutter model

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ABSTRACT

In this work a velocity-dependent friction is introduced into a depth-averaged Savage–Hutter dynamical model for shallow granular flows. The process of granular material flowing along an inclined plane and then depositing on a horizontal plane is simulated. The surface profiles and evolution of various types of energy are investigated and compared when using the standard Coulomb-type friction versus velocity-dependent friction. Interestingly, there is a small difference between the two different types of friction.

1. Introduction

Granular flows, though widely encountered in engineering and in nature (e.g., debris flows), are fundamentally different from other types of flow. Granular flows can be classified into two major categories, elastic flow and inertial flow (Campbell, 2006). In elastic flow, the mechanical properties are mainly dominated by force chains. Elastic flow can be divided into elastic-quasi-static flow and elastic-inertial flow. This sub-classification mainly depends on whether or not the shear stress is related to shear rate: for highly excited inertial flows the stress is proportional to the square of the shear rate $\dot{\gamma}$. Analysis of the internal parameters which control the mechanical properties of granular flows is very important. Shen and Sankaran (2004) suggested that the contact duration and multiple collision group size are important internal parameters in the study of dense granular flows (i.e., elastic flows). Recently a 3D discrete-element simulation of identical spheres in simple shear flow was carried out to study the mean contact time and the coordination number (Ji & Shen, 2008).

Friction coefficient is an important parameter for measuring the interaction between granular flow and its boundaries, though in many studies, this parameter is simply replaced with the static friction coefficient, as the two parameters are substantially different. Besides surface roughness and contact area, many other factors affect friction, such as shear rate, coefficient of restitution, and solid concentration. An empirical formula of the friction coefficient as a function of the inertial number I, $I=\dot{\gamma}D/\sqrt{P/\rho_p}$, was proposed, where P is the normal pressure on the granular sample, and D and ρ_p are, respectively, the particle diameter and density (da Cruz, Emam, Prochnow, Roux, & Chevoir, 2005; Pouliquen & Forterre, 2002). The number I denotes the ratio of two time scales: the characteristic time of macroscopic deformation $1/\dot{\gamma}$, and the microscopic time for a particle to rearrange $D/\sqrt{P/\rho_p}$.

Many macroscopic models have been proposed to describe granular flows, including the Savage–Hutter model for simulating the motion of granular flows on a gentle slope (Pudasaini & Hutter, 2007). The main assumptions in the Savage–Hutter model are: (1) the density of the granular flow is constant; (2) the effects of side wall and erosion are ignored; (3) the granular material remains shallow during flow. Based on these assumptions, the velocity was treated as uniformly distributed along the height perpendicular to the flow direction. Moreover, the interaction between the granular matter and the bed obeys the Coulomb friction law; that is, the frictional force is proportional to the normal pressure, and the friction coefficient is constant. To validate the above assumptions, Pudasaini, Hsiau, Wang, and Hutter (2005) carried out a series of experiments to measure the velocity distribution at the surface of granular flows using particle image velocimetry (PIV) incorporating a flash technique to illuminate the surface of the granular flow. Values for the particle velocities at different parts of the surface of the granular flow were then distinguished by analyzing the images. These authors found that, although the velocity distribution at the

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onset of the granular flows was obviously influenced by the shear layer, the depth-averaged velocity distribution became adaptable as the flow of the granular body fully developed, and the difference between calculation and measurement was small (Hutter, Wang, & Pudasaini, 2005; Pudasaini & Hutter, 2007). These authors concluded that Coulomb friction was a reasonable model for the frictional force. However, other studies of granular flows (Forterre & Pouliquen, 2008; Jop, Forterre, & Pouliquen, 2005, 2006) found that the velocity of granular flows did not possess uniform vertical distribution, indicating that the Coulomb friction model was not correct. Instead, Pouliquen and Forterre (2002) introduced a velocity-dependent friction into the Savage–Hutter model, to simulate granular flow in two dimensions along a slope using the Lagrangian approach. Their results coincided with their experimental observations. This work considers it necessary to further validate the influence of friction within the Savage–Hutter model, and employs an Eulerian approach in a one-dimensional model to study the difference between the classical Coulomb friction and the velocity-dependent friction.

An advantage of the Savage–Hutter model is that the equations are simple, though simplicity limits its application since natural granular flows possess complex geometrical boundaries. Moreover, granular flows often involve various sizes of particles, and size segregation often occurs for which the Savage–Hutter model is not valid. For example, in discrete element simulations with the particulate flow code (PFC^{3D}), Zhou and Ng (2010) found that, for polydisperse particles, the corresponding velocity profiles along the height direction were quite different. Granular flows with uniform particles have a shear boundary layer close to the bed, and the velocity distribution profile along the normal direction is parabolic. However, granular solids typically consisting of three sizes of particles flow down a slope without that same shear layer, and the velocity along the normal direction in this condition displays an almost linear distribution. Determining an appropriate size segregation in the Savage–Hutter model is another open problem that has not yet found an adequate answer.

In this work, we studied the full process of granular flow after being released on an inclined plane, simulating the flow until the material was deposited on a horizontal plane. To compare the influence of friction, both a velocity-dependent friction and a Coulomb-type friction were used and various kinds of energy were monitored. From the analysis of these results, a criterion for static deposition is presented based on kinetic energy evolution. Some differences in the evolution of energies between the two friction laws are explained, and we conclude that the velocity-dependent friction is more precise when describing granular flows.

2. The Savage–Hutter model and the friction laws

The Savage–Hutter model was first proposed to describe the motion of a finite mass of granular material flowing down a rough incline, in which the granular material was treated as an incompressible continuum (Savage & Hutter, 1989). In the original model, contact between the granular assembly and the inclined surface was considered to obey a Coulomb-type friction law. In one-dimensional flow, the effects of side wall friction and erosion on the bottom surface were ignored, and the velocity distribution profile perpendicular to the flow direction was assumed to be uniform. Thus, the conservation equations of mass and linear momentum in the flow direction $\bar{X}$ could be written as

$$\frac{\partial H}{\partial T} + \frac{\partial (HU)}{\partial \bar{X}} = 0, \tag{1}$$

$$\frac{\partial (HU)}{\partial T} + \frac{\partial (HU^2)}{\partial \bar{X}} = gH \sin\varsigma - gH\,\mathrm{sgn}(U)\tan\delta\cos\varsigma - \frac{1}{2}g\frac{\partial}{\partial \bar{X}}(K_{\mathrm{act/pas}}H^2(\bar{X},T))\cos\varsigma, \tag{2}$$

where H denotes the height of the granular flow from the free surface at position $\bar{X}$; U is the flow velocity of the granular flow in the $\bar{X}$ direction, T denotes the time of evolution, ζ is the inclination angle, δ is the bed friction angle, $K_{\mathrm{act/pas}}$ is the coefficient of active/passive earth pressure, and g is the acceleration of gravity. Eq. (2) can be re-written in dimensionless form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \sin\varsigma - \mathrm{sgn}(u)\tan\delta\cos\varsigma - \varepsilon K_{\mathrm{act/pas}}\cos\varsigma\frac{\partial h}{\partial x}, \tag{3}$$

where u denotes the dimensionless velocity in the $\bar{X}$ direction, $u = U/\sqrt{gL_0}$, and t the dimensionless time $T/\sqrt{L_0/g}$, ε denotes a scale factor, defined as the ratio of the characteristic height H_0 to the characteristic length L_0, i.e., $\varepsilon = H_0/L_0$. x is the dimensionless position in the flow direction, $x = \bar{X}/L_0$, and h is the dimensionless height $h = H/H_0$.

In general, the Savage–Hutter model well simulates granular flows on a gentle slope. However, as mentioned above, the assumptions used to simplify the equations in this model limit its applicability. Despite this limitation, the Savage–Hutter model has been further developed over the intervening years to solve many practical problems. For example, the model has been improved for two-dimensional (or more) complex situations so as to adapt it to natural topography (Pudasaini & Hutter, 2007). In this work, we use a velocity-dependent friction in the Savage–Hutter model, and compare our results with simulations using the original Coulomb-type friction.

An empirical friction law has been proposed (da Cruz et al., 2005; Jop et al., 2005; Midi, 2004; Pouliquen & Forterre, 2002):

$$\mu(\bar{U},h) = \mu_s + (\mu_2 - \mu_s)\left(\frac{H\beta\sqrt{gH}}{\bar{U}Dl_0} + 1\right)^{-1}, \tag{4}$$

where μ_s, μ_2, β and l_0 are material constants and $\bar{U}$ is the average flow velocity in the normal direction. A new constitutive friction law was developed, applicable to uniform dense granular flows:

$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{(I_0/I) + 1}, \tag{5}$$

where I_0 is a material constant. Substituting Eq. (5) into Eq. (3), we get

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \sin\varsigma - \mathrm{sgn}(u)\left(\mu_s + \frac{\mu_2 - \mu_s}{(I_0/I) + 1}\right)\cos\varsigma - \varepsilon K_{\mathrm{act/pas}}\cos\varsigma\frac{\partial h}{\partial x}. \tag{6}$$

By assuming a uniform velocity distribution, I can be expressed in terms of the average velocity in the vertical direction (Jop et al., 2005, 2006):

$$I = \frac{\dot{\gamma}d}{\sqrt{P/\rho_p}} = \frac{5}{2}\frac{\bar{U}}{\sqrt{gH}}\frac{1}{\sqrt{\phi\cos\varsigma}}\frac{D}{H}, \tag{7}$$

where D is particle diameter and ϕ is solid concentration. Using Eq. (7), we get the friction coefficient

$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{(2I_0H\sqrt{gH}\sqrt{\phi\cos\varsigma}/5D\bar{U}) + 1}. \tag{8}$$

By comparing Eqs. (8) and (4), the parameters β and l_0 in Eq. (4) can then be determined. In this work, we simply use Eq. (4) to replace the Coulomb friction in the Savage–Hutter model.