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A statistical study on the stress–strain relation of progressively debonded composites

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HIGHLIGHTS

- The new constitutive model of SiC/Al composites (aluminum alloy) has been developed.
- The model can be used to predict effective elastic modulus of three-phase composites.
- Stress–strain relations of progressively debonded composites are graphically predicted.

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ABSTRACT

In the study, a constitutive model of SiC/Al composites (aluminum alloy) has been developed, which can describe the evolution of interfacial debonding damage between particles and matrix. Eshelby's equivalent inclusion method and self-consistent method have been extended to three-phase composites, and the effective elastic modulus of SiC/Al composites is predicted. It is assumed that the particle, interphase and matrix are all inclusions of composites. Based on the incremental relation between the three phases, the incremental constitutive relations of composites, matrix, interphase, and particles have been obtained. Moreover, the statistical stress–strain relations of progressively debonded composites are graphically predicted and discussed.

1. Introduction

One of the most basic problems for composite materials is the prediction of effective or average macroscopic properties in terms of the properties and relative amounts of the individual materials or phases. They are very useful in assessing the performance that can be expected from formulations of composite materials. Many researches have attempted to estimate the effective elastic modulus of composites using both analytical and numerical methods [1–3]. Togho et al. [4] established an incremental damage model, which is based on Mori–Tanaka's mean field concept and can describe the evolution of debonding damage, matrix plasticity and particle size effects on the deformation and damage of composites.

Dimensions and volume fractions of the interphase between particle and matrix are comparable with those of reinforcement and matrix. Therefore, the ignorance of their existence would induce a high error in predicting the effective elastic modulus of composites. The formation of interphase is from many sources, some are formed during the chemical reaction between the particle and matrix, and others are from the intentionally introduction for some special purposes. The interphase undoubtedly affects the mechanical behaviors of composites in the engineering application [5]. Many relevant works have been conducted, and the influence of particle size on the composites was also explained from the interphase point of view. By introducing the interphase into the homogenization method, Vollenberg et al. [6], Boutaleb et al. [7], Marcadon et al. [8], Liu and Sun [9], and Zhang et al. [10] have studied the dependence of the equivalent moduli of particle reinforced composites (PRCs) on particle size.

In this paper, the analysis method for the effective elastic modulus of composites by Bian et al. [11] has been extended to three-phase composites by introducing an interphase between particle and matrix. In the present investigation, the modified model, which is based on Eshelby's equivalent inclusion method and self-consistent method, can be employed to describe the influence of interfacial debonding damage between the particle and the matrix on material properties. The equivalent elastic modulus of particle reinforced composites is predicted using the constitutive equations obtained. Moreover, the stress–strain relations of progressively debonded composite are graphically predicted and discussed.

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2. Three-phase incremental damage theory

In this composite system, three phases of particle, matrix and interphase between them are involved. The interphase volume fraction f_I is related to that of particles f_P by

$$\frac{f_I}{f_P}=\left(\frac{d_P+2t}{d_P}\right)^3-1 \tag{1}$$

Here d_P and t are particle diameter and interphase thickness, respectively. For the sake of simplicity, the volume fraction of interphase is related to the surrounded particles in the following sections.

The elastic incremental stress-strain relations of an isotropic matrix, interphase and particles are as follows:

$$d\sigma^i=L_i(E_i,v_i)d\varepsilon^i. \quad i=\mathrm{M},\mathrm{I}\ or\ \mathrm{P} \tag{2}$$

where $d\sigma^i$ and $d\varepsilon^i$ are the incremental stress and strain, respectively, and $L_i(E_i, v_i)$ is the stiffness tensor. The above equations replaced i with M, I and P stand for the isotropic matrix, interphase and particles. E_i and v_i are Young's moduli and Poisson's ratios of the constituents, respectively. For the stress and strain of composite, the particles, interphase and matrix are represented with superscripts "P", "I" and "M", respectively, and those of the composite are shown by symbols without superscript.

Fig. 1 shows the state before and after incremental deformation of composites in the damage process, where a constant macroscopic stress σ and its increment $d\sigma$ are applied on the boundary of composites for studying the uniaxial stress response of composites. The debonding damage is supposed to occur between particle and interphase in this model. The states before deformation are described in terms of the volume fractions of the intact and debonded particles f_P^d. If the volume fraction of the particle debonded during the incremental deformation is denoted by df_P as shown in Fig. 1, the state after deformation is described in terms of the volume fractions of the intact and debonded particles f_P–df_P and $f_P^d+df_P$. The composite undergoing the damage process contains the intact and debonded particle, and particles to be debonded during an incremental deformation in the matrix. The initial volume fraction of interphase f_{I0} is $f_{P0}[(1+2t/d_P)^3-1]$, here d_P is particle diameter, t is interphase thickness, and f_{P0} is the initial volume fraction of particles.

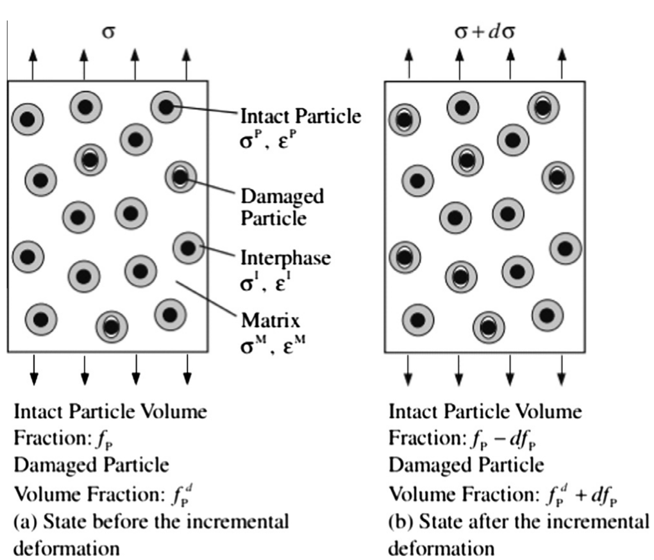

Fig. 1. The states of composite undergoing damage process before and after incremental deformation.

The model of an infinite matrix with one single particle and associated interphase in it has been chosen. The elastic modulus and strain of the three phases are E_i and ε^i (i = M, I, P). According to self-consistent method, the site of particle is displaced instead of an equivalent strain $\bar{\varepsilon}$, and its elastic modulus is E_M which is the same as that of matrix. Bian et al. [11] assumed that the matrix is in the same situation as the particles, that is, matrix and particles are both inclusions of the composite. In the present work, for the three-phase composite, it is supposed that the interphase around the particle is also the inclusion of composites. Based on the above assumptions, the elastic modulus of an equivalent matrix is E_a, which equals the average value of elastic modulus of matrix, particles, and the interphase, and can be written as $E_a=(3K_a, 2G_a)$. The equivalent strain is $\bar{\varepsilon}^a$ due to this equivalent matrix, and the equations can be written as follows:

$$\overline{L_P}\cdot\varepsilon^P_{mn}=(1-f_P-f_P^d-f_I-f_I^d)\cdot L_M\cdot\varepsilon^M_{mn}+f_P\cdot L_P\cdot\varepsilon^P_{mn}+f_I\cdot L_I\cdot\varepsilon^I_{mn} +f_I^d\cdot L_I\cdot\varepsilon^{I-d}_{mn} \tag{3}$$

$$\varepsilon^i_{mn}=A_i\cdot\overline{\varepsilon_{mn}}, \quad i=M,I,P \tag{4}$$

where ε^i_{mn} is the strain of the i-phase inclusion, which denotes the strain of matrix, particles or interphase. $\overline{\varepsilon_{mn}}$ is the average strain of an equivalent material, of which the modulus is E_a. The parameter A_i is strain concentration factor. Based on the solution by Eshelby [12], the strain concentration factor can be expressed as $A_i=[I+S_iC^{-1}(C_i-C)]^{-1}$. Here, $C=f_{P0}\cdot C_P+f_{I0}\cdot C_I+(1\text{-}f_{P0}\text{–}f_{I0})\cdot C_M$. The parameters C, C_i and S_i are all isotropic tensors, and can be written as C (3K, 2G), C_i (3K_i, 2G_i), S_i (α, β). Based on the definition by Walpole [13], the parameter A_i is also isotropic tensor and $A_i=\frac{C}{C+S_i(C_i-C)}$, that can be decomposed into two parts[14]:

$$A_i=\left(\frac{K}{K_i}\rho_1,\frac{G}{G_i}\rho_2\right) \tag{5}$$

where K and G are the average bulk and shear modulus of composite, respectively, and are given as follows:

$$K=f_{P0}\cdot K_P+f_{I0}\cdot K_I+(1-f_{P0}-f_{I0})\cdot K_M \tag{6}$$

$$G=f_{P0}\cdot G_P+f_{I0}\cdot G_I+(1-f_{P0}-f_{I0})\cdot G_M \tag{7}$$

where $f_{P0}(=f_P+f_P^d)$ and $f_{I0}(=f_I+f_I^d)$ are initial volume fractions of particles and interphase, respectively, and:

$$\rho_1=\frac{K_i}{K(1-\alpha)+\alpha K_i},\rho_2=\frac{G_i}{G(1-\beta)+\beta G_i} \tag{8}$$

$$\alpha=\frac{3K}{3K+4G},\beta=\frac{6(K+2G)}{5(3K+4G)} \tag{9}$$

By Eqs. (3) and (4), the effective elastic modulus of particle is obtained as:

$$\overline{L_P}=\frac{1}{A_p}[(1-f_{P0}-f_{I0})\cdot L_M\cdot A_M+f_P\cdot L_P\cdot A_P+f_I\cdot L_I\cdot A_I+f_I^d\cdot L_I\cdot A_I^d] \tag{10}$$

where $\overline{L_P}=(3\overline{K_P},2\overline{G_P}),\overline{K_P}$ and $\overline{G_P}$ are effective bulk modulus and shear modulus of particles, respectively.

By Eqs. 4, 5, and 10, the relation between strains of particles and matrix can be obtained as follows:

$$d\varepsilon^P_{mn}=A_P\cdot[(1-f_{P0}-f_{I0})\cdot L_M\cdot A_M+f_P\cdot L_P\cdot A_P+f_I\cdot L_I\cdot A_I+f_I^d\cdot L_I\cdot A_I^d]^{-1} \cdot[(1-f_{P0}-f_{I0})\cdot L_M+f_P\cdot L_P\cdot\tfrac{A_P}{A_M}+f_I\cdot L_I\cdot\tfrac{A_I}{A_M}+f_I^d\cdot L_I\cdot\tfrac{A_I^d}{A_M}]\cdot d\varepsilon^M_{mn} \tag{11}$$

where the term $d\varepsilon^M_{mn}=(d\varepsilon^M_{kk},d\varepsilon'^M_{mn})$ is matrix strain, and the term $d\varepsilon^P_{mn}=(d\varepsilon^P_{kk},d\varepsilon'^P_{mn})$ is particle strain.