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A generic model for investigation of arching action in reinforced concrete members

Hamid Valipour [a,*], Nima FarhangVesali [b], Stephen Foster [a]

[a] Centre for Infrastructure Engineering and Safety (CIES), School of Civil and Environmental Engineering, The University of New South Wales (UNSW), Sydney, Australia
[b] Centre for Built Infrastructure Research (CBIR), School of Civil and Environmental Engineering, University of Technology (UTS), Sydney, Australia

HIGHLIGHTS

- ▶ Development and application of a simple and efficient generic model.
- ▶ Nonlinear analysis of reinforced concrete members including arching action.
- ▶ Effect of support stiffness and concrete compressive strength on arching action.
- ▶ Implication of concrete softening on the FE analysis of arching action.

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ABSTRACT

A generic nonlinear compound frame model is developed and applied for investigation of arching action in reinforced concrete beams. This model takes account of geometrical and material nonlinearities including cracking and crushing of the concrete and yielding of reinforcing steel. Further, a non-local model is adopted to resolve the numerical sensitivity associated with compressive softening of concrete. The effect of support stiffness is incorporated into the formulation by a set of nodal rotational and translational springs that, respectively, represent the flexural and axial stiffness of the supports. The developed model is verified by available experimental data and advanced continuum-based FE models and it is shown that the generic model can predict the ultimate loading capacity and load–displacement response of reinforced concrete beams and one-way slabs with reasonable accuracy. Lastly, a parametric study is undertaken and effects of end supports stiffness and concrete compressive strength on development of arching action is investigated.

1. Introduction

The transverse deflection of reinforced concrete (RC) beams and slabs is associated with cracking of the section in the tensile zone and change of the neutral axis position, which causes axial extension in the member. If this extension is prevented by some axial restraint, such as that provided by end span columns and adjacent beams in framed structures, a compressive force is induced in the beam (arching action), which can dramatically increase the flexural capacity. The development and magnitude of this arching action depends significantly on the beam axial stiffness as well as stiffness and strength of end supports provided for the beam [1].

Arching action is one of the primary mechanisms that can improve the collapse resistance of structures, especially during extreme loading scenarios associated with column loss. Moreover, the enhancing effects of arching action in RC slab decks has been recognised and implicitly implemented in empirical methods adopted by some bridge design standards to improve the efficiency of design [2–4].

The main focus of studies on membrane action in reinforced concrete structures is related to floor and bridge deck slabs [5–11] and only a few researches have studied arching action of reinforced concrete beams within framed structures [1,12–14]. The available experimental data for laterally restrained beams and slabs show that the ultimate loads for flexural failure can exceed those predicted by normal design methods and in some cases collapse loads of between 3 and 4 times of those predicted by yield-line theory have been observed [5]. These tests also revealed the importance of geometrical non-linearities on the load-carrying capacity and the adverse effect of large secondary moment induced by coupling of the axial force and the large deflection on the membrane action [15].

Simple elastic and plastic analyses ignore the effect of arching action and these methods consistently underestimate the ultimate loading capacity of laterally restrained RC beams. Accordingly, several attempts have been made to develop analytical methods that

* Corresponding author. Address: School of Civil and Environmental Engineering, The University of New South Wales (UNSW), Sydney, Kensington, NSW 2052, Australia. Tel.: +61 2 9385 6191.
E-mail address: H.Valipour@unsw.edu.au (H. Valipour).

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properly capture the enhancing effect of arching action and predict the failure load with reasonable accuracy. These methods take advantage of either axial force-bending interaction diagram or principles of plastic analysis and a linear elastic-perfect plastic material model, which is inconsistent with the real quasi-brittle behaviour of concrete [1,16–18]. Further, existing analytical methods do not take account of end support stiffness and, accordingly, their application is limited to beams and slab strips with both ends fixed or pinned.

In addition to analytical methods, finite element models have been employed to capture the arching action. However, due to numerical complexities associated with geometrical nonlinearities and concrete crushing and cracking such as spurious mesh sensitivity, only a few finite element models have been successful in capturing the arching action of RC members [13,19,20]. The available continuum-based FE models are time demanding and not practical for undertaking parametric studies, whereas generic frame models offer the accuracy and efficiency required for such studies and is the focus of this paper [9,20].

In this paper a generic compound model is formulated and employed to investigate arching action in RC beams. The model takes account of geometrical nonlinearity as well as concrete and reinforcing steel material nonlinearities. Softening of concrete under compression is taken into account with a non-local integral model employed to resolve the numerical sensitivity associated with compressive softening of the concrete. Furthermore, effect of support stiffness is incorporated into the model by a set of nodal nonlinear springs to represent the flexural and axial stiffness of the supports and, accordingly, the proposed generic model can be considered as an extension of the formulation proposed by Valipour and Foster [20]. The developed generic model is verified by available experimental data and advanced continuum-based FE models. The verified analytical tool is then employed for a parametric study where the effect of stiffness of end supports and concrete compressive strength on the enhancing effect arching action for an RC beam subject to a concentrated load at the mid-span is investigated.

2. Formulation of generic model

2.1. Strain–displacement compatibility equations

In this model, the Navier–Bernoulli assumptions, $\varepsilon_x = \varepsilon_r - y\kappa$ (Fig. 1a) are adopted, and using the principle of virtual force and integration by parts for the simply supported compound element, shown in Fig. 1b, the strain-deformation compatibility equation (without nodal springs) is obtained as

$$\bar{\mathbf{q}}' = \int_0^l \bar{\mathbf{b}}^{\mathrm{T}}[x, w(x)]\mathbf{d}(x)\mathrm{d}x, \tag{1}$$

where

$$\bar{\mathbf{b}}[x, w(x)] = \begin{bmatrix} -1 & 0 & 0 \\ -w(x)/2 & x/l - 1 & x/l \end{bmatrix}, \tag{2}$$

In Eq. (1), $\bar{\mathbf{q}}' = [\bar{q}_1' - \bar{q}_1'' \quad \bar{q}_2' \quad \bar{q}_3']^{\mathbf{T}}$ is the generalised deformation vector of the compound model excluding the nodal springs and $\mathbf{d}(x) = [\varepsilon_r \quad \kappa]^T$ is the section generalised strain vector.

2.2. Equilibrium equations and constitutive law of material

Adopting the small slope assumption, $\sin\theta \cong \tan\theta \cong \theta = w'$, the equilibrium equations for the free body of *Ax* (Fig. 2) leads to the matrix representation

$$\bar{\mathbf{D}}(\mathbf{x}) = \mathbf{b}[\mathbf{x}, \mathbf{w}(\mathbf{x}), \theta(\mathbf{x})]\bar{\mathbf{Q}} + \bar{\mathbf{D}}^*(\mathbf{x}), \tag{3}$$

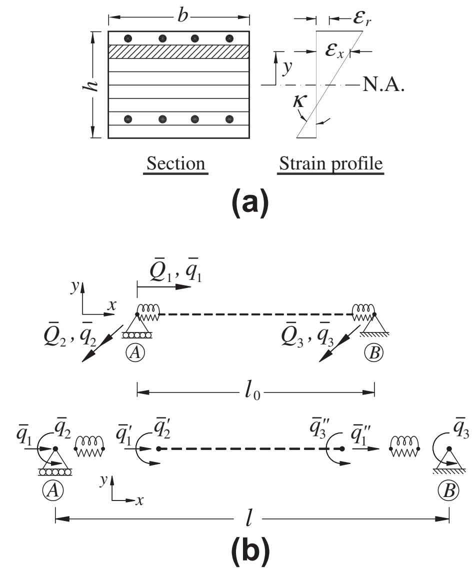

Fig. 1. Outline of the (a) strain distribution over the section depth and (b) generic compound element (without rigid body modes).

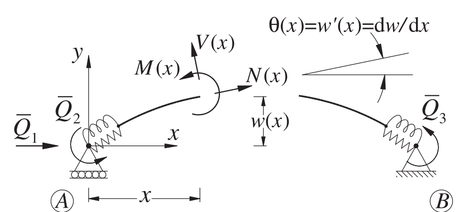

Fig. 2. Equilibrium in the simply supported configuration and free body diagram of *Ax*, after deformation (system without rigid body modes).

where

$$\mathbf{b}[x, w(x), \theta(x)] = \begin{bmatrix} -1 & -\frac{\theta(x)}{l} & -\frac{\theta(x)}{l} \\ -w(x) & \frac{x}{l} - 1 & \frac{x}{l} \end{bmatrix}, \tag{4}$$

and $\bar{\mathbf{Q}} = [\bar{Q}_1 \quad \bar{Q}_2 \quad \bar{Q}_3]^{\mathbf{T}}$ denotes the nodal force vector (without rigid body modes, Fig. 1b), $\bar{\mathbf{D}}(\mathbf{x}) = [N(x) \quad M(x)]^{\mathbf{T}}$ is the section internal force vector and $\bar{\mathbf{D}}^*(\mathbf{x})$ is the section internal force vector due to the member load.

In this study a composite Simpson integration method together with piecewise parabolic interpolation of the curvature are employed to calculate the rotation $\theta(x)$ and the displacement $w(x)$ along the member length (Fig. 1) and to update the geometry [21].

Decomposing the total strain ε_x into its elastic ε_{ex} and inelastic ε_{px} components, the total secant constitutive law is expressed by $\sigma_x = E_e(\varepsilon_x - \varepsilon_{px})$, where E_e, is the elastic secant modulus of the theoretical unloading curve (Fig. 3).

For concrete in compression, the stress–strain law presented by CEB-FIP model code 1990 is adopted for the ascending branch, followed by a linear softening branch down to the zero stress. A linear elastic brittle failure stress–strain model is adopted for the concrete in tension (Fig. 3a). A bilinear stress–strain relationship with linear unloading is used for reinforcing steel (Fig. 3b). With regard to the softening branch of the stress–strain relationship for concrete in compression, and the potential for lack of objectivity over softening regime, a non-local damage model is employed in which the concrete damage parameter, ω, is calculated from the non-local strain, $\bar{\varepsilon}$, and is denoted by $\omega(\bar{\varepsilon})$ (see Fig. 3a).

Substituting the total secant constitutive law in the section equilibrium equations, yields [20]