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# Scattering of thermal waves by a heterogeneous subsurface spheroid inclusion including non-Fourier effects

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ABSTRACT

In this paper, a general solution for the temperature field generated by the modulated heating an opaque, semi-infinite solid containing a heterogeneous subsurface spheroid inclusion was presented. Based on the non-Fourier equation of heat conduction, multiple scattering of the thermal waves from the subsurface spheroid inclusion in the semi-infinite solid was investigated by using the wave function expansion method and the virtual image method. The thermal waves were generated at the surfaces of the semi-infinite solid by modulated, ultra-short laser pulses, and the boundary condition for the inclusion was conductive. The effects of the geometrical and physical parameters on the temperature distribution were analysed. Meanwhile, numerical results for the temperature variation on the frontal surface of the solid were calculated. The analytic method and numerical results can be used for analysis of heat conduction, thermal wave imaging, the physical inverse problem and evaluation of subsurface defects in solid materials.



## 1. Introduction

With the development of non-destructive testing and evaluation techniques, infrared non-destructive testing techniques are widely used as efficient tools for the non-destructive evaluation of the thermal characteristics of opaque materials because of its non-contact and convenient operation [1–3]. The thermal waves are wave motion with dissipative natures. Recently, it is required by the thermal wave technology to have high sensitivity for detecting internal microstructures in solids [4]. The primary goal of these techniques is to reconstruct the size, depth and physical properties of the subsurface defect from the measurement of the surface temperature, which requires the development of quantitative models for thermal waves scattered from different subsurface defects to compare with the experimental data, and it is very important to determine the surface temperature accurately [5].

Many studies of the temperature distribution in solids containing different types of defects have been reported. Regular geometries, such as planes, spheres, and cylinders, can be analysed with a high degree of accuracy and have been investigated in detail by several groups [6–11]. These studies are mainly based on Fourier's law of heat conduction. Fourier's law requires an assumption that the propagation speed of a thermal disturbance in a medium is infinite; therefore, it is unsuitable for the study of micro-scale heat transfers [10]. When the surfaces of opaque materials are heated by modulated, ultra-short laser pulses, the heat propagation in the materials have wave characteristics [12], and non-Fourier effects should be taken into account. Therefore, the results will be more accurate if a thermal wave model that includes non-Fourier effects is used to calculate the temperature rather than the Fourier heat conduction equation.

This research was performed to analyse the multiple scattering of thermal waves, including non-Fourier effects, between a heterogeneous spheroid inclusion and the surface of a semi-infinite solid. Using a wave function expansion method, an analytical solution for the temperature at the frontal surface of the semi-infinite structure was obtained. Then, the influence of the geometrical and physical parameters on the surface temperature was analysed. Finally, the photothermal technique, which is based on the thermal wave model including non-Fourier effects, can be used for non-destructive testing and evaluation, and the non-destructive detection of thermal waves, including non-Fourier effects, can determine the safety and reliability of structures.

## 2. Thermal wave modelling and its solution

The geometry of the problem is shown in Fig. 1. The structure is assumed to be semi-infinite and optically opaque solid. A spheroid inclusion of radius $a$ is embedded beneath the surface with a depth

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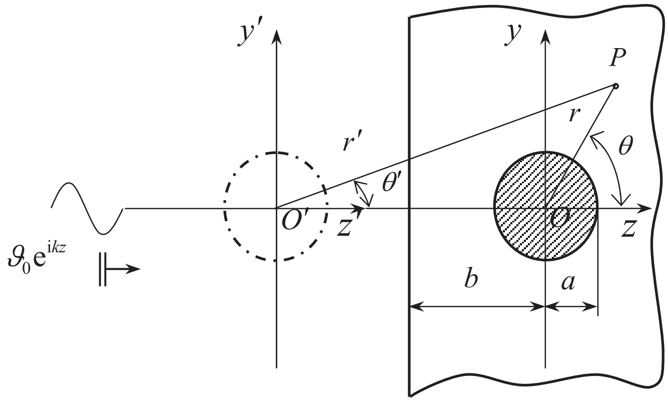


**Fig. 1.** Laser pulse heating the specimen.

of $b$. The thermal conductivity and thermal diffusivity of the structure are given by $\lambda$ and $D$, and those of the spheroid inclusion are $\lambda_0$ and $D_0$. The sample surface is heated by an extended light beam modulated at a frequency $f$ that produces planar thermal waves. The thermal waves can be described using the following hyperbolic equation of heat conduction, which is based on non-Fourier heat conduction [10]:

$$\nabla^2 T = \frac{1}{c^2}\frac{\partial^2 T}{\partial t^2} + \frac{1}{D}\frac{\partial T}{\partial t} \tag{1}$$

where $T$ stands for the temperature in solid, $\nabla = \mathbf{i}\partial/\partial x + \mathbf{j}\partial/\partial y + \mathbf{k}\partial/\partial z$ is the Hamiltonian operator, $D = \lambda/\rho c_p$ is the thermal diffusivity, $\lambda$, $c_p$, $\rho$ are the thermal conductivity, specific heat, and mass density of the solid, respectively. $c = \sqrt{D/\tau}$ is the velocity of thermal waves, and $\tau$ is the relaxation time of thermal waves.

According to the Fourier series expansion law, the unstable periodic temperature solution of Eq. (1) can be described by using the following relation:

$$T = T_0 + \mathrm{Re}[\vartheta \exp(-i\omega t)] \tag{2}$$

where Re is the real part of an imaginary number, $T_0$ is the environmental temperature, $\vartheta$ is the temperature amplitude of complex variables, and $\omega = 2\pi f$ is the circular frequency of the incident wave.

By substituting Eq. (2) into Eq. (1), the follow equations can be obtained:

$$\nabla^2\vartheta + \kappa^2\vartheta = 0 \tag{3}$$

$$\kappa = \left(\frac{\omega^2}{c^2} + \frac{i\omega}{D}\right)^{1/2} = \alpha + i\beta \tag{4}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is the Laplace operator, $\kappa$ is the complex wave number, and $\alpha$, $\beta$ are the wave number of thermal waves and attenuation coefficient, respectively. The corresponding normalized parameters are given as follows.

$$\alpha = \sqrt{\frac{1}{2}\left(\sqrt{\frac{\omega^4}{c^4} + \frac{\omega^2}{D^2}} + \frac{\omega^2}{c^2}\right)} = \sqrt{\sqrt{\frac{1}{4}k^4 + \frac{1}{\mu^4}} + \frac{k^2}{2}} \tag{5}$$

$$\beta = \sqrt{\frac{1}{2}\left(\sqrt{\frac{\omega^4}{c^4} + \frac{\omega^2}{D^2}} - \frac{\omega^2}{c^2}\right)} = \sqrt{\sqrt{\frac{1}{4}k^4 + \frac{1}{\mu^4}} - \frac{k^2}{2}} \tag{6}$$

where $k = \omega/c$ is the wave number of the incident wave without a thermal-diffusion effect. If $c \to \infty$, then $\alpha \to \sqrt{\omega/2D} = \sqrt{\pi f/D} = 1/\mu$;

$\kappa = \alpha + \beta i \to (1+i)\sqrt{\omega/2D} = (1+i)/\mu$ .

Using the limit form of the complex wave number, the hyperbolic equation of heat conduction, which is based on non-Fourier's law of heat conduction, is changed into the parabolic equation of heat conduction, which is based on the classical Fourier's law of heat conduction where the velocity of the thermal waves is infinite.

According to Ref. [11], the scattered field of thermal waves in solids is determined by Eq. (3) and can be written in the following form:

$$\vartheta = \sum_{n=0}^{\infty}\sum_{m=-\infty}^{n} A_{mn} h_n^{(1)}(\kappa r) P_n^m(\cos\theta) e^{im\varphi} \tag{7}$$

where $h_n^{(1)}(x) = \sqrt{\frac{\pi}{2x}} H_{n+1/2}^{(1)}(x)$ is the $n$th order of the spherical Hankel functions of the first type and $H_n^{(1)}(\cdot)$ is the $n$th order of the Hankel functions of the first type, $P_n^m(\cdot)$ is the associated Legendre function, and $A_{mn}$ are the mode coefficients of the scattered waves.

## 3. Thermal wave field in solids

It is assumed that the sample surface is heated by an extended light beam modulated at the frequency $f$. Then, plane thermal waves are generated. When the thermal waves impinge on the semi-infinite edge, they are scattered by the spheroid inclusion. Next, the outgoing wave that were scattered from the spheroid inclusion are reflected from the surface of the semi-infinite solid.

Thus, the temperature at any point in the structure can be written as the contribution of an incident wave $\vartheta_1^{(\mathrm{in})}$, a series of successive scattered waves $\vartheta_1^{(\mathrm{sc})}$, and a series of reflected waves from the semi-infinite edge $\vartheta_1^{(\mathrm{re})}$ [13]:

$$\vartheta = \vartheta_1^{(in)} + \vartheta_1^{(\mathrm{sc})} + \vartheta_1^{(\mathrm{re})} \tag{8}$$

According to the wave theory image method, the successive scattered waves reflected by the structure surface can be calculated as if they were emitted by an spheroid inclusion 'image' located in front of the surface and travelling in the opposite direction:

$$\vartheta = \vartheta_1^{(in)} + \vartheta_1^{(\mathrm{sc})} + \vartheta_2^{(\mathrm{rc})} \tag{9}$$

where $\vartheta_2^{(\mathrm{rc})}$ is the wave scattered by the spheroid inclusion 'image'.

### 3.1. Incident waves at the frontal surface

As shown in Fig. 1, the thermal wave propagates in the $x$-direction. The incident wave in the actual semi-infinite solid can be expanded as a series of spherical waves in terms of the ordinary spherical Bessel functions: