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A convex approach to a class of non-convex building HVAC control problems: Illustration by two case studies

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ABSTRACT

In this paper, a convexification approach is presented for a class of non-convex optimal/model predictive control problems more specifically applied to building HVAC control problems. The original non-convex problems are convexified using a convex envelope approach. The approach is tested on two case studies: a benchmark building HVAC system control problem from the literature and control of a hybrid ground-coupled heat pump (HybGCHP) system. For the first application, convexified model predictive control was used and results were compared with fuzzy and adaptive control results. For the HybGCHP system, convexified optimal control was applied and the results were compared with dynamic programming based optimal control. In the first case superior performance was observed over the corresponding fuzzy and adaptive control results from the literature. For the HybGCHP system the associated convexified optimal control gave almost global optimal results in terms of responses and cost criteria. The suggested method is especially useful for optimal building HVAC control/design problems which include non-convex bilinear or fractional terms. Since a polynomial expression can be recursively expressed as a system of bilinear equations, the proposed method, in principle, can be applied to all systems where polynomial non-convexities exist.

1. Introduction

In the context of energy-efficient buildings HVAC control has gained increasing attention in recent years. Especially, future worries about the shortage of fuel sources and the requirement of reduction in greenhouse gas emission levels necessitate building HVAC control systems to be more efficient. HVAC devices and the building itself are often modeled using physical principles of heat transfer, thermodynamics and fluid mechanics. These models usually include nonlinearities and non-convexities which pose difficulties for controller design. Although it is not the aim to list all nonlinearities and non-convexities encountered in building HVAC control systems, among them the bilinear and fractional terms are the most dominant ones. An example of a bilinear term in building HVAC applications is the mass flow rate times temperature. The coefficient of performance of a heat pump, which is the ratio of the thermal power delivered to the building over the electrical power used is an example of a fractional expression in building HVAC applications.

Once the building HVAC control system includes a bilinear or fractional term, the underlying system is a nonlinear system from the control point of view and it is a non-convex system from the optimization point of view. If the controller design is based on optimization (like optimal or model predictive control), then the controller design task basically involves solution of a non-convex optimization problem. It is very hard to solve non-convex control problems over longer control periods due to a large number of decision variables and the possibility of divergence. Even in case of a solution, a global minimum cannot be guaranteed. Existing solvers cannot handle non-convex optimization problems with a large number of decision variables. The simplest solution to such a non-convex control problem is to linearize the model around some operating point and using linear optimization. However, this leads to the risk of designing a non-working controller on the real system or a working controller with suboptimal results. As a result, linearization is not desirable and should be avoided whenever alternative controller design options are available.

A challenging HVAC control application where bilinear/fractional terms exist is the control of ground-coupled

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Variables	**Description**	**Unit**
COP	coefficient of performance	[–]
c_e	electricity price	Euro/(kWh)
c_g	gas price	Euro/(kWh)
C_a	heat capacity	kJ/K
H_t	global heat transfer coefficient of the building envelope	W/K
J	total energy-use cost	Euro
k	conductivity	W/(mK)
N_c	control horizon length	[–]
N_p	prediction horizon length	[–]
r_{fg}	grout region inner radius	cm
r_{gs}	grout region outer radius	cm
t	time	s
$\dot{P}_{ch}$	electrical power used by chiller	W
$\dot{P}_{gb}$	electrical power used by gas boiler	W
$\dot{P}_{hp}$	electrical power used by heat pump	W
$\dot{P}_{pc}$	electrical power used by passive cooler	W
$\dot{Q}_c$	cooling load demand	W
$\dot{Q}_{ch}$	thermal power extracted from the building through active cooling	W
$\dot{Q}_{ext}$	thermal power extracted from ground	W
$\dot{Q}_{gb}$	thermal power supplied to the building by gas boiler	W
$\dot{Q}_{gain}$	internal gains	W
$\dot{Q}_h$	heating load demand	W
$\dot{Q}_{hp}$	thermal power supplied to the building by the heat pump	W
$\dot{Q}_{inj}$	thermal power injected to ground	W
$\dot{Q}_{net}$	net thermal power injected to ground	W
$\dot{Q}_{pc}$	thermal power extracted from the building through passive cooling	W
T	temperature	°C
α_g	diffusivity; exponent	m^2/s; [–]
η_{gb}	gas boiler efficiency	[–]

Subscripts

a	ambient air
aff	affine
c	convex
ch	chiller
f	fluid
fr	fractional
g	grout
gb	gas boiler
hp	heat pump
i	indoor air; inlet
max	maximum
min	minimum; minimize
p	pipe
pc	passive cooler
s	soil

Abbreviations

DP	dynamic programming
GCHP	ground-coupled heat pump
HVAC	heating, ventilation and air conditioning
HybGCHP	hybrid ground-coupled heat pump
NMPC	nonlinear model predictive control
OC	optimal control
POD	proper orthogonal decomposition

heat pumps (GCHP) and hybrid ground-coupled heat pump systems combined with low-exergy heat emission systems [1–10]. The attractivity of such systems comes from having the potential to reduce the primary energy use related to space heating and cooling by 70% compared to conventional heating and cooling systems [11]. For GCHP systems with vertical borehole heat exchangers (BHE), however, the large investment cost of the borefield represents a major bottleneck. This explains the trend toward compact, hybrid GCHP systems which combine smaller borefields with supplementary heating or cooling devices such as gas-fired boilers and chillers. Although the design of a compact HybGCHP system is often driven by cost considerations to limit the drilling cost without compromising thermal comfort in the building, sometimes other reasons may also lead to HybGCHP systems, such as limited drilling area for boreholes, the specific ground characteristics, regulation or too high imbalance of the thermal load.

De Ridder et al. [12] and Verhelst [5] used mathematical model-based control methods for HybGCHP systems, which allow global optimization. However, they are based on some simplifications and/or some unrealistic assumptions introduced during the controller design. For example, De Ridder et al. [12] used dynamic programming. Dynamic programming is a powerful method since it is a closed-loop, global optimal control algorithm. However, the model used by De Ridder et al. [12] for dynamic programming is a very simple first-order model for the ground mean temperature. The chosen control time step for the system is one week, which is very long since typical control actions for buildings may require control time steps in the order of minutes or hours. Moreover, the realization of the designed controller requires the measurement of the underground field temperature, for which measurement may be either difficult or non-accurate. As a result, the approach of De Ridder et al. [12] involves both some modeling simplifications and a hard-to-realize implementation. Verhelst [5] applied a linear optimal control method. The simplification made in this work is that the coefficients of performance (COP) for heat pump and chiller were taken to be constant, in contrast to being functions of source and sink temperatures. COP values were taken to be constant to avoid a non-convex optimization problem, which cannot be solved over a horizon of a couple years especially when short control time steps are considered. Although a mathematical model-based optimal control was considered, the simplifications of taking the mentioned COPs as constant values without a formal justification restricts the work of Verhelst [5]. Moreover, the model used for control and emulator was the same, which neglects the impact of model mismatch and therefore limits the generality of the approach followed.

The objective of this paper is to present and illustrate a convex relaxation method for a class of non-convex optimal control and non-convex model predictive control problems applied to two case studies, among which the control of a HybGCHP system to minimize the total energy cost is a special case. The convex relaxation method is based on the use of convex envelopes for bilinear and fractional terms. The convex envelope of a function is the largest convex function majorized by that function. Approximation of the non-convex terms by their convex envelopes will transform the optimization problem to an approximate problem which is convex and for which the global minimum can be found, if the problem is feasible. In convex optimization problems, a local minimum is a global minimum. Although the calculation of a convex envelope for a general multi-variable function is non-deterministic polynomial-time hard, there exist analytical formulas for a bilinear function or a rational function of two variables. Moreover, it is recursively possible to represent a polynomial non-convexity as a system of bilinear equations and hence the proposed convexification method, in principle, can be used for all systems where polynomial non-convexities exist.