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# Simulation of heat and moisture transfer in a multiplex structure

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ABSTRACT

We present a 1D numerical model of heat, steam, and water diffusion across a wall, consisting of several layers of different material. The model is a system of coupled diffusion equations for material temperature, vapour pressure, and water concentration in material pores, accounting for vapour condensation and water evaporation. The system of nonlinear partial differential equations (PDEs) is solved numerically using the finite difference method. The primary objective of modelling is the simulation of the long-term behaviour of the building wall moisture distribution under the influence of seasonal variations in atmospheric air temperature and humidity.



## 1. Introduction

Heat and mass transfer in a capillary porous media has been a subject of theoretical and experimental studies for a long time [1]. This phenomenon has a particular importance in civil engineering [2], where most of the construction materials have a capillary porous structure. A large difference in the air temperature inside and outside a building during the cold season at sufficiently high air humidity can cause moisture condensation in the building's interior walls. An increasing moisture concentration usually leads to an increasing thermal conductivity in the wall material and, therefore, to a decreasing thermal resistance in the building envelope. Moreover, liquid water condensate inside a multilayer wall can cause the structural damage and deterioration of wall materials.

In his book, K.F. Fokin [2] proposed the phenomenological equations for modelling simultaneous heat and moisture transfer in a multilayer wall. He considered two zones in the wall: the dry zone and the wet zone with movable boundaries between zones. In the dry zone, the moisture could move through the material pores in the form of water vapour diffusion only. In the wet zone containing liquid water, it is supposed that the dynamic equilibrium is maintained between water and vapour, so the vapour pressure in the wet zone is equal to the pressure of the saturated vapour at a given temperature. The change of water concentration occurs because of water diffusion, vapour condensation, and water evaporation. Fokin's book [2] also contains several illustrative and tutorial numerical examples of model solutions based on the finite difference method using the explicit Euler scheme.

One of the first rigorous numerical studies of the problem of coupled heat and moisture transfer was performed by Y. Ogniewicz and C.L. Tien [3]. They considered the model, which is similar to the Fokin model, but excluded the diffusion of condensate and investigated its steady state solutions. One of the early works studying numerically the transient regimes in building constructions is reference [4]. Here and in later numerical studies using the finite difference method, Refs. [5–7] were considered to have more sophisticated models of moisture transfer. Unfortunately, detailed models are related to the introduction of new parameters, which are often known very approximately.

Recently, there have been more studies, which examine the energy balance of a building as a whole in relation to the inner and outer environments. In this case, the model of heat, air and moisture transfer (HAM model) in a wall is a part of the general model, so it should be reasonably simple to avoid large computational cost [8,9]. A new application of the HAM model could be mentioned for the computational studies of the moisture freezing effect on wall material structures, which is important for regions with cold climate [10,11].

The main goal of this paper is not the development of an original model for the phenomenon, but the presentation of a sufficiently precious and stable numerical algorithm for the solution of the model invented by Fokin and its implementation in a contemporary multi-processor PC. The Fokin model is a phenomenological model based on simple, intuitive physical principles, and it is still widely

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used in civil engineering practice in Russia for the estimation of the long-term thermal and moisture behaviour in building envelopes.

The paper is organized as follows. In Section 2, we present the mathematical formulation of the Fokin model. Section 3 contains the numerical scheme and some peculiarities of the algorithm for a numerical solution. In Section 4, we demonstrate a numerical example, and several conclusions are presented in Section 5.

## 2. Mathematical formulation of the model

### 2.1. Model parameters

For definiteness, in this paper, we will consider a three-ply wall. The thickness of the wall layers is denoted as $d_1$, $d_2$, and $d_3$ [m], respectively. Let the first layer be the external (outward building) layer and the third layer be the internal (inward building) layer of the wall. We will consider the fluxes of heat and moisture in the direction traverse to the wall only, the direction along the $x$ axis. The point $L_0 = 0$ is the outer edge of the external layer, the points $L_1 = d_1$ and $L_2 = L_1 + d_2$ are the interface points between the layers, and the point $L_3 = L_2 + d_3$ is the outer edge of the internal layer.

The material of each layer is characterized by the following parameters: density $-\rho[\mathrm{kg/m^3}]$; isobaric heat capacity – $c[\mathrm{kJ/(kg \times K)}]$; coefficients of thermal – $\lambda[\mathrm{kJ/(h \times m \times K)}]$; vapour – $\mu[\mathrm{g/(h \times m \times Pa)}]$; and hydraulic – $\beta[\mathrm{g/(h \times m \times \%)}]$ conductivity. The parameters $\rho$, $c$, and $\mu$ are supposed to be constant in the considered range of temperature $T$[K] and moisture volumetric concentration $\omega$[%], while the other parameters depend on $\omega$. For a given material, the dependencies $\lambda(\omega)$ and $\beta(\omega)$ used in the calculations are polynomial interpolations of the experimental measurements. Under equilibrium conditions at constant temperature, the relation between the air humidity $\varphi$ and moisture $\omega$ is determined from an experimentally measured sorption isotherm $\omega = o(\varphi)$.

The external layer of the wall contacts with the atmospheric air, and the internal layer contacts with the building interior air. The air temperature $T_{\mathrm{ex}}(t)$, $T_{\mathrm{in}}(t)$, and humidity $\varphi_{\mathrm{ex}}(t)$, $\varphi_{\mathrm{in}}(t)$, are given functions of time $t$[h]. The heat exchange between the wall and air is determined by coefficients $\alpha_{\mathrm{ex}}$ and $\alpha_{\mathrm{in}}[\mathrm{kJ/(h \times m^2 \times K)}]$. The steam exchange at the wall sides is determined by coefficients $\gamma_{\mathrm{ex}}$ and $\gamma_{\mathrm{in}}[\mathrm{g/(h \times m^2 \times Pa)}]$.

### 2.2. Heat conductivity

The transverse heat transfer in each wall layer is described by the heat conduction equation for the material temperature $T(t, x)$:

$$c\rho \frac{\partial T(t,x)}{\partial t} = \frac{\partial}{\partial x}\left(\lambda(\omega)\frac{\partial T(t,x)}{\partial x}\right) + Q(T,\omega) \tag{1}$$

The source term $Q$ takes into consideration the latent heat of vapour condensation and water evaporation (the water–ice transitions are not included in the model).

At the wall borders and layer interfaces, there are imposed boundary and conjugation conditions:

$$-\lambda(\omega)\frac{\partial T(t,x)}{\partial x}\bigg|_{x=L_0} = \alpha_{\mathrm{ex}}(T_{\mathrm{ex}}(t) - T(t,x))|_{x=L_0}; \tag{2}$$

$$\begin{gathered} T(t, L_i - 0) = T(t, L_i + 0), \\ \lambda(\omega)\frac{\partial T(t,x)}{\partial x}\bigg|_{x=L_i-0} = \lambda(\omega)\frac{\partial T(t,x)}{\partial x}\bigg|_{x=L_i+0}, \quad i = 1, 2; \end{gathered} \tag{3}$$

$$-\lambda(\omega)\frac{\partial T(t,x)}{\partial x}\bigg|_{x=L_3} = \alpha_{\mathrm{in}}(T(t,x) - T_{\mathrm{in}}(t))|_{x=L_3}. \tag{4}$$

### 2.3. Vapour and liquid water conductivity

It is supposed that the water in the wall material can be in three forms: water vapour (steam), mobile liquid water in material pores, and immobile absorbed water rigidly connected with material skeleton. The concentration of absorbed water $\omega$ depends upon the local air humidity in pores $\varphi(t, x)$ and is assumed to be equal to the equilibrium concentration $o(\varphi)$. If $\varphi < 1$, the material contains only steam and absorbed water, and the mass of the absorbed water in unit volume of material is equal to $0.01\omega\rho$.

The air humidity $\varphi$ is defined as ratio $\varphi = e/E(T)$, where $e$ [Pa] is the partial pressure of the water vapour and $E(T)$ is the pressure of the saturated vapour at air temperature $T$. For the calculation of $E$, we use the approximation formula:

$$E(T) = \{ \tag{5}$$

The function $\varphi(t, x)$ is supposed to be the continuous function of both variables. So, at every $t$, one can define the subset $V_t \subseteq (L_0, L_3)$, $V_t = \{x : \varphi(t, x) < 1\}$, and the complementary subset $W_t = (L_0, L_1)/V_t$.

In subset $V_t$, the moisture moves in the form of steam only, and its motion is governed by the vapour conduction equation:

$$\xi(\omega)\rho\frac{\partial e(t,x)}{\partial t} = \mu\frac{\partial^2 e(t,x)}{\partial x^2}. \tag{6}$$

(Thermo-diffusion of steam, that is, the diffusion induced by the temperature gradient, is not included in the model.) Here, the parameter $\xi$ defines the ‘vapour capacity’ of the material and can be defined by using the equation:

$$\xi(\omega) = \frac{do(\varphi)}{d\varphi}. \tag{7}$$

As noted above, Eq. (6) is defined in the subset $V_t$. If the point(s) $L_0$ and/or $L_3$ are the boundary points of $V_t$, then the boundary conditions of the convective exchange of steam between the air in material pores and the surrounding air are imposed similar to (2) and (4) by replacing $T$ by $e$, $\lambda$ by $\mu$, and $\alpha$ by $\gamma$. If any of the interface points $L_1, L_2$ belong to $V_t$, then the conjugation condition, assuming the continuity of the vapour pressure and flux, is imposed similar to (3).

In subset $W_t$, the material pores contain liquid water together with water vapour, and it is supposed that the water and vapour keep up the dynamic equilibrium. Namely, the partial pressure of vapour in $W_t$ equals to the pressure of saturated vapour, $e(t, x) = E(T(t, x))$ for all $x \in W_t$. To estimate the volumetric concentration of liquid water $w$[%], we propose the following formula

$$w(t,x) = \omega(t,x) - o(1), \tag{8}$$

where $o(1)$ is the maximal conception of absorbed water, corresponding to $\varphi = 1$.

The diffusive motion of liquid water is described by equation

$$10\rho\frac{\partial w(t,x)}{\partial t} = \frac{\partial}{\partial x}\left(\beta(\omega)\frac{\partial w(t,x)}{\partial x}\right) + \mu\frac{\partial^2 E(T)}{\partial x^2} \tag{9}$$

Here, the numerical coefficient ‘10’ appears because of different dimensions of parameters (kg in $\rho$, g in $\mu$ and $\beta$, and % in $w$). The second term on the right hand side of the equation responds to vapour condensation or water evaporation depending on its sign. To avoid extra model complications, the condition of water impermeability at border points of $W_t$ is accepted as:

$$\frac{\partial w}{\partial x}\bigg|_{\partial W_t} = 0. \tag{10}$$

If any of the points $L_1, L_2$ is the inner point of $W_t$, then the condition of continuity for the water concentration and water flux is