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# Calibration of building thermal models using an optimal control approach

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ABSTRACT

The prediction of a building's thermal behaviour within a short time horizon is necessary in many energy management applications. A numerical model can serve this purpose provided a good accuracy is obtained through a suitable calibration procedure. The paper deals with a model calibration procedure based on short-time on-site and weather measurements. It builds upon optimal control theory: an adjoint model is introduced to derive the gradient of a least squares cost function at a low computational cost. Two problems are solved. The first one is a non-linear model training problem. It consists in identifying the main influencing parameters of the system of partial differential equations that form the tendency model. The second problem is a linear identification problem that consists in identifying the unknown internal gains. This second problem can be solved in real-time in a continuous monitoring process. Both problems are solved within the same framework and same tools, illustrating the efficiency of the optimal control tools in this context. We give simulation results that show the performance of the calibration procedure under uncertainties on input parameters.



## 1. Introduction

The accurate prediction of the evolution of the thermal state of a building within a time horizon of a few hours is of great importance in energy management applications [1,2]. Examples of such techniques include a wide range of approaches such as artificial intelligence-based techniques [3], model predictive control or demand-response applications. Model predictive control consists in computing optimal heating or cooling strategies by taking into account the future evolution of the state of the building under forecast weather or use conditions [4,5]. Demand response strategies in smart grids consist in adjusting energy demand at the end-user level to reduce the overall demand thus resulting in end-user customer bill savings, increase of electricity market stability and of electricity supply reliability [6].

Such a prediction can be obtained using a numerical model that implement the most predominant phenomena explaining the evolution of the thermal state. However, modelling simplifications and uncertainties concerning building characteristics such as geometry or material properties usually lead to discrepancies between the model predictions and the real performance. The desired model response can be obtained if the internal parameters of the model are calibrated using on-site measurements and model identification methods [7,8].

This paper deals with an identification methodology used for the calibration of a building energy model based on short-term measurements of indoors and outdoors temperature, heat consumption and total solar radiation. In order to be compatible with a large scale deployment the model described here was designed to rely on very simple end-user provided data such as floor area, envelope surface, windows surface, orientation and composition of the wall. The calibrated model performance was assessed under large uncertainties on these data.

There exists a wide literature dealing with the identification of building models. Regression techniques like ARX or ARMAX have been used with success for the prediction of temperature evolution in buildings [9]. Several works report on the use of neural networks for model training (see [10–12] for instance). This kind of approaches is often referred to as black-box modelling approaches, even if some attempts to introduce physical knowledge blur the frontiers of the classification, like in [13] for instance. Their main disadvantage is the long measurement periods needed to derive the desired model.

The so-called "grey-box" modelling approaches combine physical considerations and experimental data. Madsen and Holtst [14]

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**Nomenclature**

| | |
|---|---|
| $s$ | transparent glazing total surface ($m^2$) |
| $S$ | total envelope surface ($m^2$) |
| $C$ | zone heat capacity ($J\,K^{-1}$) |
| $C_w$ | heat capacity per volume of envelope ($J\,K^{-1}\,m^{-3}$) |
| $C_a$ | heat capacity per volume of air ($J\,K^{-1}\,m^{-3}$) |
| $k$ | heat conductivity of envelope ($W\,K^{-1}\,m^{-1}$) |
| $h_i$ | convective exchange coefficients ($i = 1, 2$) ($W\,K^{-1}\,m^{-2}$) |
| $\Phi$ | total solar radiation ($W\,m^{-2}$) |
| $Q$ | internal heat gains (W) |
| $R$ | air mass exchange rate ($m^3\,s^{-1}$) |
| $\mathcal{W}$ | supplied electric heating energy (W) |
| $\Gamma_0$ | solar input coefficient for the zone |
| $\Gamma_2$ | solar input coefficient for the external envelope |
| $T_e$ | outdoor air temperature (K) |

use the maximum likelihood principle to determine the parameters of a continuous-time model with two time constants. Identification of various linear and non-linear models are compared in [15]. Braun and Chaturvedi [16] propose a two-step identification procedure for training the parameters of a simple thermal network model. The use of techniques such as genetic algorithms has been reported [17] for the estimation of simplified building models. The main difficulty in all those approaches consists in finding the model's level of complexity that gives the best compromise between sufficient representation of reality and information embedded in the data. Some attempts, including use of model reduction techniques, have been made to systematically determine that optimum [18,19].

The identification methodology is derived in a continuous framework based upon optimal control theory. The model equations are the standard partial differential equations for heat diffusion. The main model parameters are calibrated during a training step through an identification procedure. The identification problem is set as a quadratic cost functional minimization problem and the optimal solution is obtained through the introduction of an adjoint system of equations. We also suggest to solve a second identification problem to determine, in real-time, the internal gains that are not caught by the model. This works like a real-time model performance indicator in a monitoring process.

The use of this continuous optimal control framework has several advantages. First, the problem and solution algorithm is derived in a continuous setting independently of time and space discretization issues. Any discretization software can thus be used and the algorithms can be implemented with a high-level programming architecture. The second advantage concerns computational cost. The cost function gradient can be obtained in a very fast way by solving the adjoint model, even in the case when the unknowns are time-dependent functions. It is thus possible to solve in real-time a load-estimation problem that enables to determine unknown internal gains. This gives a way to monitor in real-time the accuracy of the model. Last, the identification procedure scales-up very nicely for multizone buildings models with increasing geometrical complexity.

The paper is organized as follows. We first describe the modelling assumptions and the derivation of a non-dimensional system of equations that makes appear the identifiable parameters. We then present the framework used to calibrate the model. For his, the adjoint model is introduces and the Levenberg–Marquardt algorithm is used to minimize a regularized quadratic cost functional. A second optimization problem is suggested to estimate in real-time the internal gains not caught by the model. We finally give some numerical results that show the relevance of the proposed model

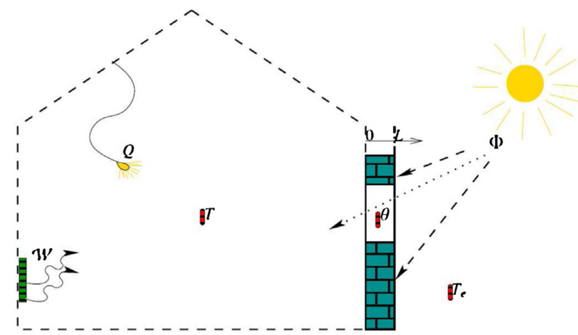


**Fig. 1.** A thermal zone and the various heat transfers.

and training strategy to forecast the temperature evolution within a short time horizon, depending on the quality of short term and local weather conditions. The sensitivity of the model under important input data uncertainties is assessed.

## 2. Model derivation

Although the proposed methodology applies to large scale buildings consisting of multiple rooms or areas, we present it here at the scale of a single building zone. We adopt here standard zone modelling assumptions and we define a thermal zone as a subvolume of the building, possibly comprising several rooms, in which the supplied heating power is controlled by a single regulation system and in which ambient air temperature is supposed to be uniform (Fig. 1).

### 2.1. Modelling assumptions

Energy transfers in a building are either related to air mass transfer phenomena or to heat exchanges between the various components of the envelope. These two kinds of energy transfers have dynamics of very different time constant so that it is necessary to let them appear separately in the model equations [14]. Only the external envelope is taken into account so that interactions between adjacent zones are neglected.

The evolution of the temperature $T$ inside the zone is governed by a standard ordinary differential equation:

$$\begin{cases} C\dfrac{dT}{dt} = \Gamma_0 s\Phi + C_a R(T_e - T) + h_1 S(\theta(0,t) - T) + Q + \mathcal{W} \quad t \in [0, \tau] \\ I.C. \text{ (initial conditions)} \end{cases} \tag{2.1}$$

In this equation, $T$ (K or °C) is the temperature inside the zone and $t \in [0, \tau]$ is the time variable, $C$ ($J\,K^{-1}$) is an overall heat capacity representing the quantity of heat that can be stored within the building, including the effects of furniture and other non-envelope components, while $C_a$ ($J\,K^{-1}\,m^{-3}$) is the heat capacity of the air volume.

The first term in the right hand side corresponds to solar heat gains: $\Phi = \Phi(t)$ ($W\,m^{-2}$) is the total solar radiation, $s$ ($m^2$) is the transparent glazing total surface, and $\Gamma_0$ is a non-dimensional coefficient which represents the portion of the total solar radiation that effectively enters the zone. This coefficient depends on the orientation of the outside façade and the position of the sun.

The second term of this equation represents the infiltration or ventilation exchange between the zone and the outdoor environment, $R$ ($m^3\,s^{-1}$) being the mean air mass exchange rate. The outdoor air temperature is denoted $T_e$ and the third term corresponds to convective heat transfers between the inside surface of the wall at temperature $\theta(0, t)$ (see below) and the zone. Convective transfers depend on the convective exchange coefficient $h_1$ ($W\,K^{-1}\,m^{-2}$) and the total envelope surface $S$ ($m^2$). Last, $Q = Q(t)$