Energy and Buildings 76 (2014) 176–184

Contents lists available at ScienceDirect

Energy and Buildings

journal homepage: www.elsevier.com/locate/enbuild

Model of unsteady heat exchange for intermittent heating taking into account hot water radiator capacity

Agnieszka Lechowska*, Artur Guzik

Department of Environmental Engineering Cracow University of Technology ul. Warszawska Cracow 24 31-155, Poland

ARTICLE INFO

Article history:
Received 10 July 2013
Received in revised form
28 November 2013
Accepted 21 February 2014

Keywords:
Intermittent heating
Transient heat exchange
Control volume method

ABSTRACT

Intermittent heating is one of the methods leading to savings in energy consumption. The intermittent heating system can work with reduced power or it can be completely cut off when the rooms are not occupied. At the beginning of the cut-off mode, the radiator remains warm for a specific period of time, due to its thermal capacity. This capacity is not negligible and should be considered for buildings with light or very light structures. This paper outlines a mathematical model of unsteady heat exchange in rooms with light wall structure with intermittent heating. The air heat balance of a given room takes into account the room air capacity, hot water radiator capacity, heat transfer through walls, ceiling, floor and windows as well as air infiltration. Reasonable accuracy between calculation and measurement results has been achieved. With known air and radiant temperatures, air humidity and velocity, thermal comfort indices predicted mean vote (PMV) and predicted percentage of dissatisfied (PPD) were evaluated in order to verify how thermal comfort changes during radiator cut-off mode. The satisfactory convergence between measured and calculated internal air temperatures has been achieved.

1. Introduction

Reducing energy consumption in buildings is an important environmental and economic issue. One of the methods leading to such savings is intermittent heating in which the central heating system can work in continuous heating mode at a constant set-point temperature as well as in switch-off mode with a night time and/or weekend reduced set-point temperature [1–5]. During continuous heating mode, thermal comfort indices PMV and PPD [6] are set at a constant level and no energy is saved. During switch-off heating mode, a building's energy consumption is lowered, although inside thermal comfort is also decreased.

Not all buildings are constructed similarly. They can have different structures, i.e. with different thermal heat capacities: very light, light, medium, heavy and very heavy elements [1]. In this paper the attention is focused on modelling the heat dynamics of the indoor air temperature in a light building heated by a low surface temperature hot-water radiator in a moderate climate during the heating season. In the building with a very light or light structure, radiator capacity should be taken into account.

The paper is organised as follows. The model of unsteady state heat exchange in buildings is introduced in Section 2. The calculation data and test room is described in Section 3. The measurement and calculation results are presented in Section 4, and finally, conclusions with discussion are given in Section 5.

2. Mathematical model

An energy balance of room internal air can be written as [7–9]:

$$V_a \rho_a c_a \frac{dT_a}{dt} = \dot{Q}_r + \dot{Q}_{gn} - \sum_{j=1}^{6} \dot{Q}_s - \dot{Q}_{win} \tag{1}$$

where:

$$\frac{dT_a}{dt} = \frac{T_{a2} - T_{a1}}{t_2 - t_1} \tag{2}$$

$$\dot{Q}_{win} = \left(U_{win} A_{win} + H_{\mathrm{inf}}\right)\left(T_{a2} - T_{sol}\right) \tag{3}$$

$$\sum_{j=1}^{6} \dot{Q}_s = \sum_{j=1}^{6} \frac{A_{sj}}{R_{\lambda sj} + R_{\alpha sj}} \left(T_{a2} - T_{sj2}\right) \tag{4}$$

In Eq. (3) H_{inf} is constant value accounting for an external air infiltration, dependent on several factors, including building height, window air infiltration rate, and total length of gaps around the

---

* Corresponding author. Tel.: +48 12 632 09 48; fax: +48 12 628 20 48.
E-mail address: alechowska@quino.wis.pk.edu.pl (A. Lechowska).

http://dx.doi.org/10.1016/j.enbuild.2014.02.062

Nomenclature

a	solar radiation absorptivity
A	area, m^2
c	specific heat capacity, J/(kg K)
C	radiator constant
I	global solar irradiance, W/m^2
m	mass, kg
n	radiator constant exponent
N	number of measurements or calculated values
$\dot{Q}$	heat flow rate, W
t	time, s
T	temperature, K
U	overall heat transfer coefficient, W/(m^2 K)
V	volume, m^3

Greek letters

α	heat transfer coefficient, W/(m^2 K)
ΔQ_{ir}	infrared radiation due to difference between the external air temperature and the sky temperature, W/m^2
ρ	density, kg/m^3

Abbreviations

MAPE	mean absolute percentage error
PMV	predicted mean vote
PPD	predicted percentage od dissatisfied
RMSE	root mean square error, K

Subscripts

a	internal air
c	ceiling
calc	calculated value
e	external
ew	external wall
f	floor
gn	internal heat gains
h	end of heating season conditions
in	inlet, supply water
iw	internal wall
meas	measured value
n	number of time steps
o	design conditions
r	radiator
s	structure, room opaque elements
sol	sol–air
w	water
win	window
α	heat convection
λ	heat conduction
0	initial
1	beginning of time step
2	end of time step

window. External air temperature T_{sol} in Eq. (3), referred to as the sol–air temperature, takes into account solar radiation and is calculated as [10,11]:

$$T_{sol} = T_e + \frac{a\,I - \Delta Q_{ir}}{\alpha_e} \tag{5}$$

Eq. (4) describes the heat conduction and convection from a boundary (room external and internal walls, floor and ceiling) to the room air.

In Eq. (1) $\dot{Q}_{gn}$ is a heat flow rate from internal heat gains from: occupants, equipment and lighting. The internal heat gains distribution is put in the model as known values [12–14].

Assuming that the room heating system consists of the low-surface-temperature horizontal hot-water radiator the supplied heat flux can be expressed as:

$$\dot{Q}_r = \frac{U_r A_r}{1 + \frac{U_r A_r}{2\dot{m}_w c_w}}\,(T_{in1} - T_{a2}) \tag{6}$$

where the overall radiator heat transfer coefficient is defined by:

$$U_r = C\,T_{a1}{}^{n}\left(\frac{T_{in1}}{T_{a1}} - \frac{T_{a1} - T_e}{T_{a1} - T_{e\_o}}\,\frac{\Delta T_{r\_o}}{2T_{a1}} - 1\right)^{n} \tag{7}$$

In Eqs. (6) and (7) radiator supply water temperature is calculated from:

$$T_{in1} = T_{in\_o} + \frac{T_{in\_h} - T_{in\_o}}{T_{e\_h} - T_{e\_o}}\,(T_e - T_{e\_o}) \tag{8}$$

Once the supply water temperature and overall radiator heat transfer coefficient based on the previous time step data are calculated, both values are inserted into Eq. (6). Inserting Eqs. (2)÷(6) into (1), the room air heat balance equation for radiator in the on-mode is given as:

$$\left(D + E + F + \sum_{j=1}^{6} G_j\right) T_{a2} - \sum_{j=1}^{6} G_j T_{sj2} = D\,T_{a1} + F\,T_{sol} + E\,T_{in1} + \dot{Q}_{gn} \tag{9}$$

where:

$$D = \frac{V_a \rho_a c_a}{t_2 - t_1} \tag{10}$$

$$E = \frac{U_r A_r}{1 + \frac{U_r A_r}{2\dot{m}_w c_w}} \tag{11}$$

$$F = U_{win} A_{win} + H_{\mathrm{inf}} \tag{12}$$

$$G_j = \frac{A_{sj}}{R_{\lambda sj} + R_{\alpha sj}} \tag{13}$$

Eq. (9) for inside air during radiator on-mode with energy balance equations comprise a set of equations solved at each time step.

During radiator off-mode, water temperature is continuously decreasing. Its energy balance equation can be expressed as:

$$(V_w \rho_w c_w + m_r c_r)\,\frac{dT_w}{dt} = -\alpha_r A_r\,(T_w - T_a) \tag{14}$$

The solution of Eq. (14), taking into account that both water and inside air temperatures are time-dependent, is given by:

$$T_{w1} = \exp(Kt_2)\left[T_{w0} - KB\,T_{a1} - KC\,T_{a2} - K\sum_{i=1}^{n-1}\left(B_i\,T_{a1,i} + C_i\,T_{a2,i}\right)\right] \tag{15}$$

where:

$$K = \frac{-\alpha_r A_r}{V_w \rho_w c_w + m_r c_r} \tag{16}$$