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A distributed parameter model for the piezoelectric stack harvester subjected to general periodic and random excitations

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ABSTRACT

Vibration-based energy harvesting using piezoelectric stacks has received increasing attention in recent years for its higher mechanical-to-electrical energy conversion capability in the d_{33} mode. However, most research on this type of harvesters is based on either the simplified one-degree-of-freedom model or transfer matrix model. This paper presents a distributed-parameter model for the multilayer piezoelectric stack transducer based on the axial vibration theory of a continuous bar. The presented analytical model is free from the assumption that the generated current is identically distributed over all the piezoelectric layers, which has been widely used by the existing models in literature. A first-order numerical model is also introduced to validate the performance of the analytical model on the prediction of voltage, current and power outputs of the harvester under different types of external excitations. Both the analytical and numerical models are firstly validated by experiments and then are used to predict the electrical responses of the stack harvester under general periodic and random excitations with different intensities. Experiment and simulation results demonstrate that the proposed distributed-parameter model has a good accuracy and reliable performance in the prediction of the electrical responses.

1. Introduction

The direct and inverse piezoelectric effects enable piezoelectric materials to possess the unique ability that converts mechanical energy into electrical energy and vice versa. Thus, piezoelectric materials have been widely used as actuators for structural vibration control [1–4], precision positioning [5–7], vibration sensors [8,9], and transducers for vibration energy harvesting [10,11]. In order to realize larger force and power outputs with a small input, hundreds of thin piezoelectric films can be mechanically layered together in series and electronically connected in parallel so as to compromise a multilayer piezoelectric stack. Multilayer piezoelectric films polarized along the longitudinal direction of the stack make the resulting harvester work in the '33' mode, in which case the mechanical-to-electrical energy conversion efficiency is 3–5 times higher than that of '31' mode [12]. The application of piezoelectric stacks as an actuator for structural vibration control have been reported in [13,14] and as a transducer for energy harvesting can be found in [15–17]. Recently, a theoretical study on the energy harvesting from a railway system by use of the piezoelectric stack transducer is carried out in [18]. The increasing applications of the piezoelectric stacks in both vibration control and energy harvesting fields attract extensive efforts on the analysis and modeling of the complex electromechanical coupling behavior of the piezoelectric material in the multilayer stacked configuration. In comparison with various well studied piezoelectric beam harvesters [19–22], the modeling and analysis of piezoelectric stack transducers and actuators are still underdeveloped. An appropriate description and interpretation of the electromechanical coupling behavior of the piezoelectric stack under various excitations are critical to the design, fabrication and optimization [23]. Furthermore, a good theoretical model is capable of reliably predicting the electrical responses including voltage, current and power outputs of the piezoelectric stack transducer.

Many efforts have been dedicated into the modeling of the multilayer piezoelectric stacks in literature, which can be categorized into four methods: single-degree-of-freedom model (SDOF), transfer matrix (TM) model, finite element (FE) model and simplified distributed-parameter (SDP) model. In the SDOF model firstly proposed by Goldfarb and Celanovic [1], the piezoelectric stack is simply considered as an equivalent mass-spring-damper system [24]. Feenstra et al. [15] integrated the SDOF model into a backpack to harvest energy from human motion and experimentally demonstrated that the model can predict the power output with an error less than 12%. The approximate SDOF model is easily implemented and has good accuracy for most applications, nevertheless, it still has some disadvantages. For instance,

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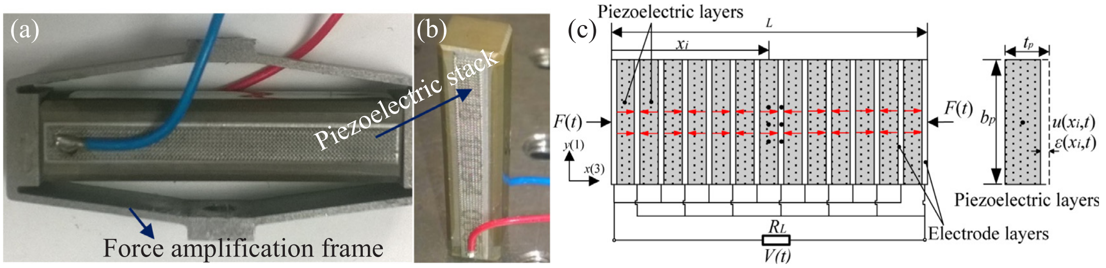

Fig. 1. (a) The piezoelectric stack transducer with a force amplification frame, (b) piezoelectric stack, and (c) electro-mechanical coupling model.

the SDOF model is based on the assumption that all the piezoelectric films electrically connected in parallel have the same current and voltage outputs, while it cannot prove this assumption due to the fact that it cannot give the prediction of each piezoelectric layer. Furthermore, SDOF system only presents single peak in a bode plot, which is discordant with the experimental results having more than one peaks [25] and the theoretical fact that there should be infinite peaks in the frequency response for a distributed parameter system [26].

The transfer matrix method is employed to analyze the dynamic characteristic of piezoelectric stack transducers with the assumption that the mechanical and electrical boundary conditions at the surfaces of each layer are continuous. Zhu et al [27] used the transfer matrix method originally for multibody systems to describe the dynamic characteristics of the piezoelectric stack actuator and experimentally validated the model. They further established the TM model of the piezoelectric stack with the consideration of bonding layer to investigate the influence of layering or stacking process on the dynamic performance [28]. Lately, Zhang et al. [23] proposed a simplified TM model by considering the whole piezoelectric stack as an equivalent homogenous bulk to facilitate the derivation of analytical solutions. The TM model can effectively describe the electromechanical dynamic response of the piezoelectric stack under the assumption that the quarter wavelengths of the vibration modes of interest are far larger than the thickness of the piezoelectric layer. The TM model is efficient to predict the electromechanical response of the piezoelectric stack since it only needs one transfer matrix for the entire stack. This method has also been applied to model a d31-mode longitudinal piezoelectric transducer recently [29].

Finite element (FE) model is also experimentally proven to be a valuable tool for the evaluation of a multilayer-stacked piezoelectric actuation/transduction system [30]. A simple distributed-parameter model is proposed and validated by FE analysis in [31] based on the assumption that the displacement across each layer was uniformly distributed. By following that, a general mathematical model is developed for considering the additional mass, damper and spring coupling with the piezoelectric actuator [32]. However, the assumption is ineffective for those vibration modes whose wavelength is close to or less than the thickness of the piezoelectric layers. Furthermore, the methods mentioned above are used to model the dynamic responses of the piezoelectric stack under a harmonic input with single excitation frequency, which cannot sufficiently simulate the real case of general periodic and random excitations. Therefore, a complete analytical model is necessary for the dynamic analysis of the stacked piezoelectric transducer under general periodic and random excitations.

This paper presents a distributed-parameter model for the multilayer piezoelectric stack transducer based on the axial vibration theory of an elastic bar. The analytical solutions to the electromechanical coupling dynamic problem of the piezoelectric stack are provided under harmonic, general periodic and random excitations. The proposed analytical model is firstly validated by experiments and then is used to predict the electrical current, voltage and power outputs under different load cases. The obtained analytical solutions are compared with the numerical simulation results obtained from the first-order numerical model. Different from existing models, the distributed-parameter analytical model does not need the assumption that the currents are identical in all the piezoelectric layers. In return, the results obtained from the presented model provide a theoretical evidence for such assumption.

2. The distributed-parameter model of the multilayer piezoelectric stack

To further increase the power output of the piezoelectric stack transducer, both extension and compression frames have been specially designed to transform and synchronously amplify the external force to the piezoelectric stack. A piezoelectric stack transducer with an extension frame is shown in Fig. 1(a). Fig. 1(b) and (c) illustrate the picture and an equivalent electromechanical model of the piezoelectric stack, respectively. This section presents the derivation procedure of the equations of motion of the piezoelectric stack in the force amplification frame [33–35]. In the stack, the piezoelectric layers with the total number of p are polarized along the x (3) direction and serially superimposed together with the electrode layers connected in parallel, as shown in Fig. 1(c). The geometric size of each piezoelectric layer is defined by $a_p \times b_p \times t_p$ and the cross section of the stack is denoted by A. The effect of electrode layers is ignored in this study considering that their thickness and density usually are far less than those of the piezoelectric layers. Thus, the overall length of the stack can be simplified as $L = pt_p$. The motion of the piezoelectric stack in the force amplification frame directly exposed to external loads can be modeled as the axial vibration of a continuous bar with free-free boundary conditions. The axial force $F(t)$ at the two ends of the stack, transferred by the frame from the external loads, are equal in value but opposite in direction. The axial displacement and electric potential at arbitrary position x of the stack are denoted by $u(x, t)$ and $\varphi(x, t)$ at time t. Thus, the strain response and electric field of the piezoelectric stack can be expressed as

$$S_3(x,\ t) = \frac{\partial u(x,\ t)}{\partial x},\quad E_3(x,\ t) = -\frac{\partial \varphi(x,\ t)}{\partial x} \tag{1}$$

For the sake of concision, the independent variables x and t in the parentheses to indicate the dependent mechanical and electric variables are dropped hereafter. The stress T_3 and electric displacement D_3 can be attained from the constitutive equations of piezoelectric material for the '33' mode transducer as following

$$T_3 = c_{33}^E S_3 - e_{33} E_3 \tag{2a}$$

$$D_3 = e_{33} S_3 + \varepsilon_{33}^s E_3 \tag{2b}$$

where $c_{33}^E = 1/s_{33}^E$, $e_{33} = d_{33}/s_{33}^E$, and $\varepsilon_{33}^s = \varepsilon_{33}^T - \left(\frac{d_{33}^2}{s_{33}^E}\right)$, and s_{33}^E, d_{33} and ε_{33}^T are the compliance constant, piezoelectric strain and dielectric constants. The superscripts 'E' and 'T' claim the corresponding parameters are measured at the constant electric field and stress, respectively.

The energy method is used to derive the equations of motion of the piezoelectric stack transducer. Assume the density of the piezoelectric material is ρ. The kinetic energy and potential energy of the dynamic system of the continuous bar can be written as