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Automatic determination of LQR weighting matrices for active structural control

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ABSTRACT

This paper presents a method for the automatic selection of weighting matrices for a linear-quadratic regulator (LQR) in order to design an optimal active structural control system. The weighting matrices of a control performance index, which are used to design optimal state-feedback gains, are usually determined by rule of thumb or exhaustive search approaches. To explore an easy way to select optimal parameters, this paper presents a method based on Bayesian optimization (BO). A 10-degree-of-freedom (DOF) shear building model that has passive-base isolation (PBI) under the building is used as an example to explain the method. A control performance index that contains the absolute acceleration, along with the inter-story drift and velocity of each story, is chosen for the design of the controller. An objective function that contains the maximum absolute acceleration of the building is chosen for BO to produce optimal weighting matrices. In the numerical example, a restriction on the displacement of the PBI is used as a constraint for the selection of weighting matrices. First, the BO method is compared to the exhaustive search method using two parameters in the weighting matrices to illustrate the validity of the BO method. Then, thirty-three parameters (which are automatically optimized by the BO method) in the weighting matrices are used to elaborately tune the controller. The control results are compared to those for the exhaustive search method and conventional optimal control, in terms of the control performance of the relative displacement, absolute acceleration, inter-story-drift angle, and the story-shear coefficient of each story. The damping ratio for each mode, and the control energy and power are also compared. The comparison demonstrates the validity of the method.

1. Introduction

Over the last a couple of decades, the number of passive-base-isolated buildings has markedly increased. Such demand in Japan has been increasing significantly after the Kobe earthquake in 1995. Nowadays, passive-base isolation (PBI) is widely used in high-rise buildings to protect properties and people inside [1].

The active structural control (ASC) strategy has also been studied to improve the control performance since around 1990. This strategy is now being widely used in large-scale buildings all over the world [2].

Many methods have been proposed for the design of a control system for ASC. Among them, the linear-quadratic regulator (LQR) is one of the most commonly used methods, and has been extensively investigated [3–15]. The LQR designs a state-feedback gain by minimizing a performance index that contains a weighted state and control input. Loh at el. conducted an experiment using a real-scale active tendon, and showed the effectiveness of the LQR for a real-scale structure [3]. Sedegh et al. compared LQR to PD/PID controllers in a high-rise building application [4]. Chu et al. also conducted an experiment for tuned-mass damper (TMD) structures considering a time delay in a control action [5].

While the state in a performance index is usually defined by a relative displacement and relative velocity, some studies chose a performance index from different viewpoints. Other definitions included elastic and kinetic energy [6,7], absolute acceleration [8], or inter-story drifts [9]. On the other hand, Miyamoto et al. [10] and She et al. [11] applied the equivalent-input-disturbance (EID) approach to ASC. The configuration of an EID-based system contained state feedback and a

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state observer, and the LQR method was used to design the gains of both.

The weighting matrices in a performance index provide a large degree of freedom in the design. Fuji at el. considered the influence of those matrices of a performance index containing only absolute acceleration for a model with a single degree of freedom (DOF) [16]. However, most buildings have multiple DOFs, and it is important to select weighting matrices for a performance index that contain other items in addition to absolute acceleration. Since the selection of weighting matrices affects the performance index, it is crucial to choose them to minimize the index and also to yield prescribed control specifications (such as overshoot, rise time, settling time). The selection is largely determined by experience, and is essentially a trial and error process [17]. One reason is that control specifications do not have explicit relationship with the weighting matrices. It is well known that the weighting matrix of state variables is related to system error and the weighting matrix of control input variables is related to control effort, but it is not clear how a change in an entry of the weighting matrices affects the control specifications. Iteration is usually used to repeat the selection of the matrices and the design of LQR. This results in a high computational expense. So, how to select those matrices has been a challenge problem of the LQR design since the 1960s [18]. The key problems of the selection are

(1) how to devise a method to adjust the matrices efficiently, and
(2) what is the criterion for the adjustment of the matrices.

An exhaustive search method is computationally expensive to search for suitable weighting matrices of a state for a high-DOF model. To ease the selection of the weighting matrices, Miyamoto at el. used the same weights for both the relative displacement and velocity [10]. While this reduces the burden for the design of a state-feedback gain, the designed gain may not meet control specifications. Harvey et al. presented a cheap optimal control method [19]. It calculates the weighting matrices of a state for a performance index that only contains a weighted quadratic term of the state. Kawasaki et al. developed a method to select a controller based on the pole placement method [20]. Fujii et al. presented an ILQ (Inverse-LQ) method based on pole placement and the inverse problem [21]. However, these methods need to select damping ratio, natural frequency, and some other parameters by trial and error. Thus, a large number of parameters need to be selected for a high-DOF plant to carry out the design of a controller. Those methods have two problems: First, the resulted weights may not be optimal ones; and second, preselected parameters may not satisfy criteria for the design, such as the maximum displacement.

Methods of automatically selecting weighting matrices have also been proposed so far. For example, Trimpe et al. proposed a stochastic optimization algorithm to find the matrices that yields a minimum of the performance index [22], and Karthick et al. employed an adaptive particle swarm optimization algorithm to solve the problem of selecting weighting matrices [23]. However, they did not take control specifications into account in the optimization process. Elumalai and Subramanian explored the relationship between the algebraic Riccati equation and the Lagrange optimization technique, and tried to determine the weighting matrices based on a prescribed damping ratio and natural period for each oscillation mode [24]. Nevertheless, their method can only be applied to a low-order model, and the selection of damping ratios and natural periods requires trial and error. As shown in the above methods, an important problem with the matrix selection is how to find a systematic way to efficiently perform the trial and error.

The Bayesian optimization (BO) method, which is a nonparametric optimization approach, can be used to automatically select weighting matrices. Even if an objective function is unknown, it can be estimated by a Gaussian process. This method has been used to select a weighting matrix in [25,26] in which the objective function was set to be the value of the performance index of the LQR. One problem is that, even if the value of the performance index is small, some state responses may be very big.

In ASC for a PBI building, the suppression of both displacement and absolute acceleration is important. Note that PBI enlarges the natural period of the building. This may result in a large displacement that extends beyond the allowable range. Thus, the suppression of displacement is necessary. However, suppressing absolute acceleration not only protects the structures by reducing the story shear coefficient, but also protects people and property by preventing things such as furniture and equipment from falling over. For these reasons, optimizing a performance index that contains only the displacement or the absolute acceleration may not produce a satisfactory result, and a large number of parameters has to be tuned in order to design a satisfactory control system. It is desirable to find an easy way to select those parameters.

This paper uses the BO method for the automatic selection of LQR weighting matrices for ASC for the first time. A performance index containing the absolute acceleration, the inter-story drift, and the inter-story velocity of each story of a PBI building is optimized, and the displacement of the PBI story is required to be less than or equal to a prescribed value. This is used as a constraint on the optimization. The weighting matrices in the performance index are determined by optimizing an objective function for the absolute acceleration. The advantages of the presented method are twofold. First, a systematic way is used to guide the trial and error process to intensively search the weighting matrices. This avoids unnecessary trial and error, and eases the search for an optimal solution. Second, criteria for the adjustment of the weighting matrices are chosen to be control specifications in the time domain. This simplifies the performance assessment, thus significantly reduces computational expense.

2. Structural model and control design

This section describes the structural model and the design of a state-feedback gain. This study used a 10-DOF shear building model with a height of 250 m (Fig. 1) to illustrate the design methodology. A PBI is located under the structure and an ASC device is installed at the PBI story. Thus, the model of the structure has 11 DOFs (10 DOFs for the superstructure and 1 DOF for the PBI) (Fig. 1). In this study, an actuator was considered as a device for ASC.

The parameters of the superstructure and the PBI are as follows:

Mass of passive base-isolation story per square meter: 2551 kg/m^2
Damping for passive base-isolation period (ζ_0): 0.05

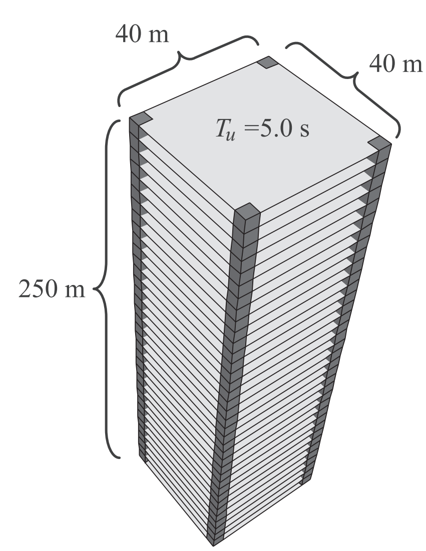

Fig. 1. Model of building.