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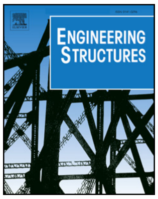

Co-rotational 3D beam element for nonlinear dynamic analysis of risers manufactured with functionally graded materials (FGMs)

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ABSTRACT

This paper considers a co-rotational beam formulation for beams, which is used for the finite element analysis of flexible risers and pipelines made of functionally graded materials. The influence of material gradation is addressed using an exponential variation of properties throughout the thickness of the pipe. Space discretization of the equilibrium equations is derived based on the Euler–Bernoulli assumptions considering two-node Hermitian beam elements which are referred to a co-rotation coordinate system attached to the element local frame of coordinates. The geometric non-linear effects of the beam are considered under large displacement and rotations, but under small-strain conditions. The deflections of the riser result from forces caused by self-weight, buoyancy, sea currents, waves, the action of floaters, seabed-structure interactions, and ship's motion. We provide numerical examples and compare our results with the ones available in the literature. In addition, applications related to practical offshore engineering situations are considered to highlight the behavior of functionally graded materials (FGMs) as compared to homogeneous risers.

1. Introduction

Flexible marine risers are used to transport oil from a wellhead on the seabed to platforms based on floating production systems, such as Spar, Floating Production Storage and Offloading (FPSO), semi-submersibles and Tension-Leg Platform (TLP). A marine riser is continuously subjected to severe environmental loading from currents, waves, and winds. For this reason, marine risers are considered one of the most critical parts of a complex offshore production system. Their failure can severely deteriorate the marine environment and interrupt the oil production, causing costly environmental recovery and lost production.

Owing to their low flexural rigidity, as compared to their axial rigidity, marine risers undergo large changes in curvature under operating loads. As a result, one needs to consider geometric nonlinearities for their structural evaluation. In some special circumstances in which marine piping undergoes (a) large deformations and (b) negligible tensile loading, its bending stiffness may become insignificant as compared to other local effects, such as cross-section ovalization [1]. In these special cases, the analysis of marine piping requires a more detailed numerical model [2,3].

Computational models based on the finite element method for static and dynamic analysis of risers involving geometric nonlinearity have been the subject of extensive research [4–12]. These models employ the classical theory of beams under finite rotations and the governing equilibrium equations are obtained from the following three basic descriptions: total lagrangian, updated lagrangian and co-rotational. The first two approaches use, respectively, the undeformed and deformed configurations, as a reference for both static and kinematic variables. Both total and updated Lagrangian formulations have been extensively used in the literature in various beam element formulations [13–17] and the only advantage of using one over the other lies in their numerical efficiency. However, under significantly large rotations, these two formulations may cause spurious stiff solutions because of the non-vectorial nature of rotation variables. To overcome such a difficulty, co-rotational formulations have been proposed in the literature [18–21] as a framework for nonlinear analysis of one-dimensional finite element models such as beam elements. In this approach, the total motion of a beam is decomposed into two parts: rigid body and pure deformational. The deformation is measured from a rotating frame attached to the element undeformed configuration, and standard linear formulation solutions are employed in the numeric incremental procedure in such a way that non-linearity is considered by the spatial rotation of the undeformed reference frame. This procedure is particularly well fitted for the finite element representation of slender pipe kinematics, such as in

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marine riser structures undergoing large three-dimensional displacements. This study considers axial, bending, and torsional deformations, but neglects transverse cross-section deformations of the pipe.

Oil exploration has shifted into more remote locations and ultra-deep water under severe operating conditions, demanding lightweight, strong, durable riser structures to reduce production costs and increase efficiency. In this regard, the use of composite materials for risers [22,23] is quite promising because, as compared to traditional materials, they offer a high stiffness to weight ratio, high tensile and fatigue strength, improved structural damping, low thermal conductivity, and good corrosion resistance. However, delamination is a critical damage mechanism in traditional composite materials under extreme mechanical and thermal loadings [24]. As a solution to this problem, material scientists from Japan proposed in the mid 1980s the concept of functionally graded materials (FGMs) [25] to design a thermal barrier capable of withstanding a temperature gradient of 1000 K across a 10-mm cross-section. These materials provide continuous graded macroscopic properties with gradual change in microstructure and are designed to take advantage of the desirable features of their constituents phases. Due to the material gradation, FGMs have no material interfaces, thus eliminating the possibility of delamination, as it is the case in traditional composite materials.

Over the last years, FGMs have been used in different industrial fields such as aerospace, nuclear science, defense, automotive and energy conversion. For instance, functionally graded (FG) plates as well as FG cylindrical shells have been reported in the literature on bending, vibration and buckling analysis [26–34].

An ideal material for marine risers would combine the best properties of metals at inner surfaces, such as toughness and machinability, and of ceramics at outer surfaces, such as high strength and temperature resistance. This would be a very effective engineering solution but requires material usage to an extent not feasible for production in our days. However, FGMs could be employed as a transition material in multi-layer riser cross-sections to prevent abrupt differences in mechanical and thermal properties and avoid debonding at certain extreme loading conditions. Because cracks are likely to initiate at interfaces and propagate into the weaker material sections, FGMs can reduce the effects of thermal, residual, or stress concentration factors [17]. Although still not available in full-scale industrial production, FGMs are man-made materials in which the method of fabrication, geometry, reinforcement location, and matrix alloy may be tailored to achieve certain desired properties.

In this study, we present a concept for marine risers that considers material gradation throughout the pipe thickness. The FGM model is isotropic with constant Poisson's ratio and its non-homogeneity arises from the variation of the Young's modulus (E) and the density (ρ) in the thickness direction according to a power law [31,35]. A beam element model based on Euler-Bernoulli kinematics is adopted, which considers both geometric nonlinearities and a mass matrix derived in closed form. In the next two sections, details of a 3D co-rotational beam finite-element formulation are given with constitutive relations associated to FGM capabilities incorporated. The formulation was implemented, and in Section 4 the results of some sample analyses are presented that illustrate the important features of graded material usage in marine risers and the effect of its non-homogeneity on the stress distribution.

2. Description of the 3D co-rotational beam model

This section presents details of the 3D co-rotational beam formulation used in the present study.

2.1. Basic assumptions

The following hypotheses are adopted to derive the behavior of the beam element:

- plane sections remain plane and perpendicular to the beam centerline after deformation (Euler-Bernoulli hypothesis);
- large displacements are allowed but only under small strain conditions;
- the material behaves elastically, and thus, element model nonlinearities result from both the spatial configuration at each instant of the analysis and the coupling of axial tension and bending deformation mechanisms; and
- under torsion loadings, the cross-section remains plane, i.e., it does not warp.

2.2. Coordinate systems

A co-rotational beam element undergoing large displacements and rotations is shown in Fig. 1. In this figure, we identify three configurations: initial configuration ($\mathscr{C}_0$), which is the element in its initial (undeformed) position at time t = 0; co-rotated configuration ($\mathscr{C}_C$), which is the element after it has been subjected to rigid body movements from its initial configuration; and deformed configuration ($\mathscr{C}_D$), which is the element in its current configuration at time t under external loading. Coordinate systems attached to the beam element at each configuration are also shown in Fig. 1 and are described as follows:

- the global frame (X_G, Y_G, Z_G) is a coordinate system describing the structure, and this system remains fixed during the entire analysis;
- the element base frame (X_0, Y_0, Z_0) is the coordinate system of the element at its initial configuration ($\mathscr{C}_0$); in this configuration, the beam element is assumed to be straight and the X_0-axis coincides with the element longitudinal direction; the other axes (Y_0 and Z_0) are set to the cross-section principal directions;
- the co-rotated frame (X_C, Y_C, Z_C) is associated with the co-rotated configuration ($\mathscr{C}_C$); the X_C-axis passes through two end nodes of the element; on this system, the entire element formulation is written; and
- the convective frame (X_D, Y_D, Z_D) is associated with the deformed configuration ($\mathscr{C}_D$); this system follows the element centerline in the deformed configuration and is attached to its center; the X_D-axis defines the cross-section orientation, and the other axes (Y_D and Z_D) are chosen along the cross-sectional principal directions.

2.3. Local beam kinematics

In this section, compatibility relations for a 3D beam model used to represent the structural behavior of risers are derived. Fig. 2 presents successive positions at two instants of analyses separated by a time increment Δt. A general point P in the riser moves from points P^t to $P^{t+\Delta t}$

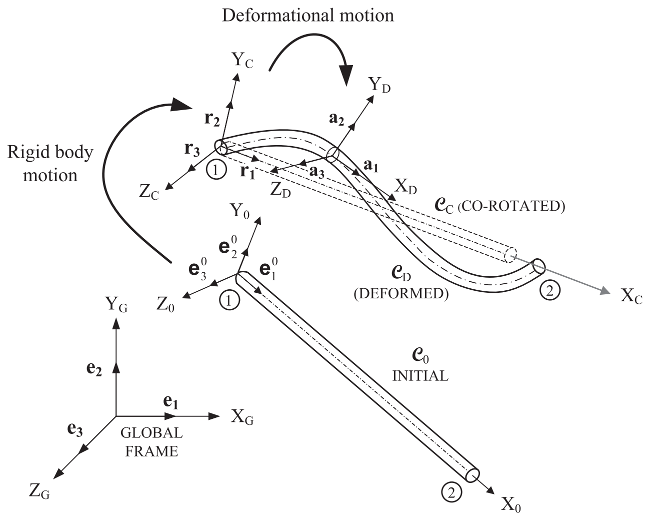

Fig. 1. Co-rotational frame of a deformed beam element.