Engineering Structures 173 (2018) 416–428

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

The importance of correlation among flutter derivatives for the reliability based optimum design of suspension bridges

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A R T I C L E I N F O

Keywords:
RBDO
Correlation
Optimization
Reliability analysis
Flutter derivatives
Random variables
Suspension bridge

A B S T R A C T

The design of long-span bridges is constrained by the uncertainty in the evaluation of flutter velocity. Among all the elements that take part in the flutter assessment, the uncertainty level in experimentally obtained flutter derivatives has the most impact. It is therefore important in the evaluation of flutter velocity to assess the uncertainty which is associated with the adopted experimental method for flutter derivatives. By using a method of coupled motion only to identify eight flutter derivations simultaneously, it is also essential to consider correlations among the points that define the full set of flutter derivatives since they are not independent from one another. In this research, an experimental campaign was carried out to obtain the statistical information of flutter derivatives and to assemble the correlation matrix. Several cases of reliability analyses were performed to illustrate the importance of considering correlation among random variables as well as the significance of uncertainty level in flutter derivatives on bridge flutter failure. Moreover, a study of Reliability Based Design Optimization (RBDO) was carried out to see the influence of correlations among flutter derivatives on the optimum designs. The RBDO of a suspension bridge was performed under probabilistic flutter constraint using Reliability Index Approach (RIA) method, and this methodology was applied to the Great Belt East Bridge.

1. Introduction

Long-span suspension bridges are highly sensitive to wind loads due to their inherent structural flexibility and low damping. Among all wind related instabilities, flutter phenomenon is one of the most important design considerations because it can lead to the collapse of the structure. For the estimation of critical flutter speed, we need to obtain the aeroelastic parameters called flutter derivatives experimentally from wind tunnel tests. However, these flutter functions contain uncertainty due to the experimental nature of the data as well as the identification method used to extract each function. In fact, some researchers such as Sarkar et al. [1] reported significant variations in the results of wind related variables obtained in different wind tunnels. They concluded that the differences in experimental results depend on the laboratory environment or operational conditions as well as the techniques used to extract the data such as number of degrees of freedom, upstream turbulence, sampling rate and time, instrumentation and the system identification method used. Consequently, the consideration of uncertainty in flutter derivatives is essential for the estimation of critical flutter velocity.

Reliability analysis of bridge flutter provides information of the probability of structural failure considering uncertainty in parameters that participate in the evaluation of flutter limit state. Several authors carried out reliability analyses of bridge failure due to flutter. Ostenfeld-Rosenthal et al. [2] performed reliability analysis of cable supported bridges by considering uncertainty in extreme wind speeds, conversion from model tests to prototype, turbulence and structural damping. Ge et al. [3] computed probability of failure due to flutter using First Order Reliability Method (FORM), in which an empirical formula was used to evaluate flutter speed in the limit state. Cheng et al. [4] carried out flutter reliability analysis using response surface method. Baldomir et al. [5] performed reliability analyses of bridge flutter considering uncertainty in experimentally obtained points that define flutter derivatives. Canor et al. [6] proposed a generalized formulation for stochastic bridge flutter in terms of random eigenvalue analysis. Caracoglia et al. [7] reported experimental errors associated with flutter derivatives and modelling simplifications regarding bridge aerodynamics. Rizzo and Caracoglia [8] studied variability and correlation of flutter derivatives in experimental tests and used polynomial chaos expansion to characterize the distribution.

However, only a few researchers have worked on the reliability analysis of long-span bridges considering correlated flutter derivatives as random variables. Matsumoto [9] reported correlations among flutter derivatives of a rectangular cylinder while Tubino [10]

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https://doi.org/10.1016/j.engstruct.2018.06.091
Received 3 November 2017; Received in revised form 23 May 2018; Accepted 25 June 2018

investigated inter-relations among flutter derivatives of streamlined box girder based on generalized quasi-steady theory. Seo and Caracoglia [11] considered the correlations between the coefficients of the polynomial functions that represent flutter derivatives and performed reliability analyses using FORM and sampling method.

In this research, reliability analyses of bridge flutter were carried out considering the correlations among the points that define flutter derivatives. These direct correlations among flutter derivative points have not been considered by other researchers to the best of author's knowledge. Moreover, the Reliability Based Design Optimization (RBDO) of suspension bridges was performed considering correlated random variables of flutter derivatives and the plate thicknesses of box girder as design variables. The shape optimization of the box girder was not considered in this study. The consideration of correlated flutter derivative points in RBDO of suspension bridges is also a novelty of this research. In order to define these correlations and to characterize statistical properties of the flutter derivative points, an experimental campaign of wind tunnel tests was carried out at the wind tunnel of the University of Coruña. The Great Belt East Bridge was used as an application example of the reliability analyses and the RBDO of suspension bridges under probabilistic flutter constraint.

In summary, the main objectives of this paper are twofold and can be summarized as follows:

- Carry out reliability analyses of bridge flutter considering correlations among the points that define flutter derivatives and study the influence of these correlations on the reliability indices of the suspension bridge.
- Perform the RBDO of the Great Belt East Bridge considering correlated random variables of flutter derivatives and analyse the significance of these correlations on the optimum designs.

2. Computation of critical flutter velocity by hybrid method

Flutter speed of long-span bridges may be calculated using hybrid method, which consists of an experimental phase of testing a bridge deck sectional model in wind tunnel and a subsequent computational phase. A detailed description of this method can be found in Jurado et al. [12]. A vibrating bridge deck under wind flow creates self-induced forces that depend on displacement vector $\mathbf{u} = (v, w, \varphi_x)^T$ and its derivative, where v is horizontal, w is vertical and φ_x is the rotational degrees of freedom of the deck as shown in Fig. 1.

The relationship between the aeroelastic forces, $\mathbf{f}_a$ and the displacement vector can be written employing a set of eighteen functions called flutter derivatives as formulated by Scanlan [13].

$$\mathbf{f}_a = \begin{Bmatrix} D_a \\ L_a \\ M_a \end{Bmatrix} = \frac{1}{2}\rho VKB\cdot\begin{pmatrix} P_1^* & -P_5^* & -BP_2^* \\ -H_5^* & H_1^* & BH_2^* \\ -BA_5^* & BA_1^* & B^2A_2^* \end{pmatrix}\begin{Bmatrix} \dot{v} \\ \dot{w} \\ \dot{\varphi}_x \end{Bmatrix} + \frac{1}{2}\rho V^2K^2\cdot\begin{pmatrix} P_4^* & -P_6^* & -BP_3^* \\ -H_6^* & H_4^* & BH_3^* \\ -BA_6^* & BA_4^* & B^2A_3^* \end{pmatrix}\begin{Bmatrix} v \\ w \\ \varphi_x \end{Bmatrix} \quad (1)$$

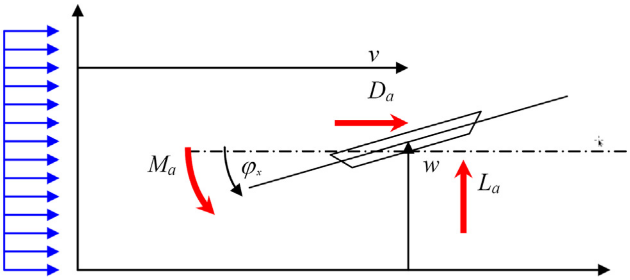

Fig. 1. Sign criterion of aeroelastic forces used in the study.

where B is the deck width, ρ is the air density, V is the acting wind speed, $K = B\omega/V$ is the reduced frequency with ω (rad/s) as the response frequency, A_i^*, H_i^* and P_i^* ($i = 1, \ldots, 6$) are the flutter derivatives obtained experimentally.

The multimodal flutter analysis was used to solve this problem, which is described in detail by some researchers such as Kastuchi et al. [14] or Chen et al. [15]. The dynamic equilibrium of a deck under aeroelastic forces, $\mathbf{f}_a$ can be written in a matrix form as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}_a = \mathbf{C}_a\dot{\mathbf{u}} + \mathbf{K}_a\mathbf{u} \quad (2)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are mass, damping, and stiffness matrices, $\mathbf{K}_a$ and $\mathbf{C}_a$ are aeroelastic stiffness and damping matrices. Eq. (2) can be solved using multi-modal analysis, which solves the flutter eigenvalue problem. The FLAS code developed at the University of Coruña [16] solves this eigenvalue problem to obtain critical flutter velocity, which was used throughout the study.

3. Experimental campaign of flutter derivatives for the Great Belt Bridge in the wind tunnel

In our previous studies of reliability analysis of bridge flutter [5] and the RBDO of long-span bridges under probabilistic flutter constraint [17,18], reliability analyses of bridge flutter were performed considering extreme wind speed at the bridge site and experimentally obtained flutter derivatives as uncorrelated random variables. The coefficients of variation for the points that define flutter derivatives were assumed to be either constant, 0.05 or 0.15 or linearly variable 0 at $V^* = 0$ and 0.15 at $V^* = 30$, where V^* is the reduced velocity defined as $V^* = V_f/(f{\cdot}B)$ being f the frequency response in Hz. These assumptions were made due to the absence of statistical data.

In order to study the correlations among flutter derivatives as well as the data dispersion in the wind tunnel, we carried out an experimental campaign of 18 wind tunnel tests at the University of Coruña to obtain flutter derivatives of the sectional model of the Great Belt East Bridge during a period of two weeks. The deck sectional model used for the study is described next.

3.1. Deck sectional model

In order to perform the experimental phase of the hybrid method, the deck cross sectional model (Fig. 2) was defined based on the geometry of the Great Belt East Bridge (Fig. 3). The important aspect of the model is to keep the geometric similarity to the real bridge deck, and the side ratio, length/width, of the model should be at least 3 so that the model can be considered as two-dimensional. Also the model should be sufficiently rigid so that it would not deform under wind loads. The sectional model of this research was scaled to 1:100 due to the dimension of the wind-tunnel test chamber. Since the turbulence of the wind tunnel was less than 0.5%, the flow was regarded as laminar. The blockage effect was not considered since the bridge sectional model height of 4 cm was small compared to the height of the test chamber of 1 m, and the test chamber was open at both top and bottom ends.

3.2. Wind tunnel tests

Aeroelastic tests were performed using free vibration method, in which the scaled deck sectional model was suspended by two pairs of horizontal springs and four pairs of vertical springs connected to load cells. After the model was displaced from its equilibrium position, it was released using pneumatic actuators. The heave and pitch movements of the model were registered using our MATLAB [20] based in-house program, first without wind several times, and then with gradually increasing wind velocity up to approximately 10 m/s. The measurement of the movements without wind is necessary because flutter derivatives are computed by subtracting the stiffness and damping matrices without wind from those with wind. For each free