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Free vibration of axially functionally graded beams using the asymptotic development method

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ABSTRACT

Axially functionally graded (AFG) beams, with variable coefficients in the governing equation, are a novel class of composites structures that have continuous variations in material properties from one component to another. In this paper, the asymptotic development method (ADM) is utilized to investigate the free vibration of uniform AFG beams with different boundary conditions. By decomposing the variable flexural stiffness and mass per unit length into reference invariant parts and variant parts, perturbation theory is introduced to obtain an approximate formula of the natural frequencies of the uniform AFG beams. The numerical results of the proposed method are confirmed by comparing the obtained results with those obtained via finite element analysis and the published literature results, the comparison reveals the proposed method yields an accurate estimate of the first three order natural frequencies of the AFG beam. Moreover, the influences of the gradient parameter and support conditions on the first three natural frequencies are discussed. The proposed analytical method is simple and efficient and can be used to conveniently analyze uniform AFG beams with arbitrary changes in the material properties along the axial direction.

1. Introduction

Functionally graded materials (FGMs) are a novel class of composites that have continuous variations in material properties from one component to another. Because of their promising mechanical and thermal properties, FGMs are typically made into a variety of structures, such as beams [1], plates [2,3] and shells [4], and are widely used in extreme engineering environments, particularly for gas turbine and aerospace engineering. Moreover, the vibration of FGMs structures has drawn considerable interest. For example, Lee et al. [5] developed an exact transfer matrix method to analyze the free vibration characteristics of FGMs beams where the material properties vary continuously along the thickness direction of the beam by a power-law distribution. Sina et al. [6] considered a new beam theory different from traditional first-order shear deformation beam theory to analyze the free vibration of FGMs beams. Li et al. [7] applied higher-order shear deformation theory to study the dynamic analysis of FGMs beams subjected to various end conditions. Avcar and Alwan [8] employed Rayleigh beam theory to research the free vibration of FGMs beams with simply supported boundary conditions. Akgöz and Civalek [9] investigated the buckling problem of linearly tapered cantilever microcolumns of rectangular and circular cross-section on the basis of modified strain gradient elasticity theory. Civalek [10] carried out the buckling, bending, and free vibration analysis of plates and columns using the harmonic differential quadrature and differential quadrature methods. Jin et al. [11] used the Haar wavelet method to compute the free vibration solutions of FGMs cylindrical shells with first-order shear deformation theory. Zhao et al. [12,13] presented a method for studying the free vibration of metal and ceramic functionally graded plates and shells using the element-free *kp*-Ritz method. Chen and his team [14–16] used the perturbation analysis method investigated the nonlinear parametric vibration of axially accelerating viscoelastic strings and beams.

It is difficult to obtain precise solutions for axially functionally graded (AFG) beams because of the variable coefficients of the governing equation, therefore, several numerical methods have been used to analyze the vibration characteristics of AFG beams. By assuming that the material constituents vary throughout the longitudinal directions according to a simple power law, Alshorbagy et al. [17] developed a two-node, six-degree-of-freedom finite element method in conjunction with Euler–Bernoulli beam theory to detect the free vibration characteristics of a functionally graded beam. Shahba et al. [18,19] used the finite element method to study the free vibration analysis of an AFG tapered beam based on Euler–Bernoulli and Timoshenko beam theory.

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Shahba and Rajasekaran [20] studied the free vibration analysis of AFG tapered Euler–Bernoulli beams using the differential transform element method. Liu et al. [21] investigated the free vibration analysis of AFG tapered Euler-Bernoulli Beams through the spline finite point method.

Alternatively, some researchers have used analytical or semi-analytical approaches to analyze the vibration characteristics of AFG beams. Huang and Li [22] presented a novel and simple approach to solve the natural frequencies of the free vibration of AFG beams. For various end supports, the governing equation with varying coefficients is transformed into Fredholm integral equations to determine the natural frequencies by requiring that the resulting Fredholm integral equation has a non-trivial solution. Huang and Li [23,24] also proposed an exact analytical method to investigate the vibration behaviors of AFG beams with arbitrary axial gradients. Hein and Feklistova [25] solved the vibration problems of AFG beams with various boundary conditions and varying cross sections using the Haar wavelet series. Li et al. [26,27] derived the exact frequency equations for the free vibration of axially exponentially graded beams with various end conditions based on Euler–Bernoulli and Timoshenko beam theory. Kukla and Rychlewska [28] proposed a new approach to study the free vibration analysis of an AFG beam, the approach relies on replacing functions characterizing functionally graded beams with piecewise exponential functions. Sarkar and Ganguliand [29] assumed that the material mass density, elastic modulus and shear modulus are simple polynomial functions along the length of the beam and obtained a fundamental closed form solution for the free vibration of AFG Timoshenko beams with uniform cross sections. Xie et al. [30] presented a spectral collocation approach based on integrated polynomials combined with the domain decomposition technique for free vibration analyses of beams with axially variable cross sections, modulus of elasticity, and mass densities. Akgöz and Civalek [31] examined the free vibrations of AFG tapered Euler–Bernoulli micro-beams based on Bernoulli–Euler beam and modified couple stress theory. Zhao et al. [32] introduced a new approach based on Chebyshev polynomial theory to investigate the free vibration of AFG beams with non-uniform cross sections.

The present study investigates the free vibration of AFG beams with uniform cross sections using the asymptotic development method (ADM). First, the governing differential equation for the free vibration of a uniform AFG beam is summarized and rewritten in a form of a dimensionless equation based on Euler–Bernoulli beam theory. By decomposing the variable coefficients into reference invariant parts and variant parts, the ADM is introduced to resolve the governing equation and an approximate formula of the natural frequencies of the AFG beams is obtained. Furthermore, the natural frequencies of a uniform AFG beams made of alumina and zirconia are analyzed considering different boundary configurations. The results are compared to those obtained via finite element analysis and the published literature results to validate the effectiveness of the ADM. The influences of the gradient parameter and support conditions on the natural frequencies of the AFG beams are also demonstrated and discussed. Finally, the conclusions are presented.

2. Problem formulation

This study considers a beam with a uniform cross-section and made of axially functionally graded materials. The beam length, width and height are denoted as L, B and H, respectively, and with a coordinate system (Oxyz) is shown in Fig. 1.

2.1. Functionally graded materials

In this paper, the material properties of the beam are assumed to vary continuously in the axial direction according to the usual power-law gradient assumption [22], so the material properties such as the Young's modulus $E(x)$ and mass density $\rho(x)$ along the beam axis are

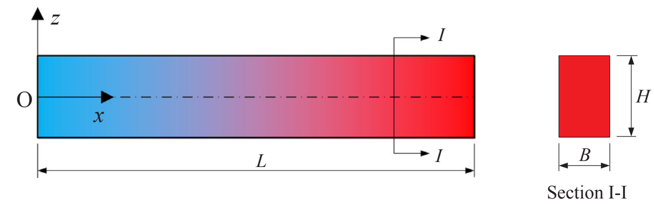

Fig. 1. The geometry and coordinate system of an AFG beam.

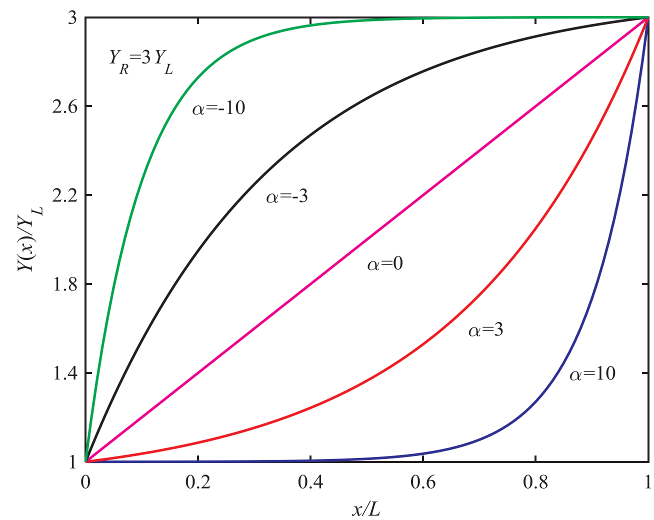

Fig. 2. Variation of the material properties defined by Eq. (1) with $Y_R = 3Y_L$.

given as follows:

$$Y(x) = \begin{cases} Y_L\left(1-\frac{e^{\alpha x/L}-1}{e^{\alpha}-1}\right) + Y_R\frac{e^{\alpha x/L}-1}{e^{\alpha}-1}, & \alpha \neq 0, \\ Y_L\left(1-\frac{x}{L}\right) + Y_R\frac{x}{L}, & \alpha = 0. \end{cases} \tag{1}$$

where Y_L and Y_R denote the corresponding material properties of the left and right sides of the beam, respectively, and α is the gradient parameter describing the volume fraction change of both constituents involved. The variation of $Y(x)$ along axis direction of the beam is shown in Fig. 2 for $Y_R = 3Y_L$.

2.2. Governing differential equation

Based on Euler-Bernoulli beam theory, the governing differential equation of the AFG beam can be written as

$$\frac{\partial^2}{\partial x^2}\left[E(x)I\frac{\partial^2 w(x,t)}{\partial x^2}\right] + \rho(x)A\frac{\partial^2 w(x,t)}{\partial t^2} = 0, \quad 0 \leqslant x \leqslant L. \tag{2}$$

where $w(x, t)$ is the transverse deflection at position x and time t; $E(x)I$ is the flexural stiffness, which depends on both Young's modulus $E(x)$ and the area moment of inertia I; and $\rho(x)A$ is the mass of the beam per unit length, which depends on both the material mass density $\rho(x)$ and cross-sectional area A.

For AFG beams, the flexural stiffness $E(x)I$ and mass $\rho(x)A$ both vary, which makes it difficult to resolve the differential equation with variable coefficients. Here, we introduce a reference flexural stiffness E_0I and a reference mass per unit length ρ_0A, these quantities will be presented and computed in Section 3. Let $E(x)I = E_0I + \overline{E(x)I}$ and $\rho(x)A = \rho_0A + \overline{\rho(x)A}$, where E_0I and ρ_0A are the invariant parts and $\overline{E(x)I}$ and $\overline{\rho(x)A}$ are the variant parts of the bending stiffness and mass per unit length, respectively. A non-dimensional equation is convenient for computational purposes. By introducing the non-dimensional space variable, defined by $\xi = x/L$, and the non-dimensional time, defined by $\tau = \frac{t}{L^2}\sqrt{\frac{E_0I}{\rho_0A}}$, Eq. (2) can be rewritten in the non-dimensional form