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Displacement and mixed fibre beam elements for modelling of slender reinforced concrete structures under cyclic loads

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ABSTRACT

In this paper, two fibre-based beam elements with enhanced capabilities to consider large displacements and rotations of slender reinforced concrete members are developed. Fibre beam elements were comprehensively used before to model the behaviour of different structural systems with great accuracy. To upsurge the use of the fibre beam elements in modelling complex reinforced concrete (RC) systems such as slender walls and columns, the elements are improved by including the second order effect. Available research from the literature related to large displacements focused mainly on modelling steel and composite members due to the limitations in their material model behaviour. Conversely, the newly developed elements introduced in this paper can precisely model RC members by accounting for their more complex nonlinear material behaviour under reversed cyclic loads. The first element is formulated using a displacement formulation, while the second element is based on a mixed approach that is computationally more complicated but numerically more efficient. Further, the adopted concrete constitutive law accounts for the effect of compression post-peak softening as well as tension stiffening and degradation under cyclic loads. Several correlation studies are presented to highlight the efficiency of the new elements in modelling slender RC structures.

1. Introduction

Fibre beam elements are frequently used to predict the nonlinear response of RC structures under static and dynamic loads. Fibre beam elements use detailed geometry and material models to obtain accurate representation of yielding and inelastic behaviour along the length of the member [1,2]. They require less storage capacity and short execution time compared to continuum elements such as membrane and solid elements. Yet, most available RC fibre beam elements do not consider second order effects. Existing second-order fiber-based elements focused mostly on steel and composite structures under monotonic loads [3–6]. Hence, in order to study the actual stability and performance of slender reinforced concrete structures under different loads, second order effects must be considered. The inclusion of second-order effects is necessary to examine slender structures such as long columns, arches, and tall buildings. In such frames, large displacements and rotations are expected to occur and the second-order effect can lead to a higher level of inelastic behaviour that must accounted for in nonlinear analysis.

The calculation of second order forces in numerical algorithms can be carried out using matrix analysis where the geometric stiffness is directly derived from the governing differential equations that consider the second-order effect of the axial force on the flexure response. This offers a simple and accurate method for the consideration of second order effects for beam-column elements. This method is also called the second-order computer program method due to the ease of its implementation in computer routines compared to other conventional methods. The geometric stiffness effect on the forces and displacements of the member usually varies between 10 and 25% depending on the ratio between the lateral and axial loads [7].

Two types of deformations are associated with the second order analysis. First, the P-δ, (called the small P-delta), where δ is related to the local deformation with respect to the chord of the element end nodes and can be considered by subdividing the element into smaller segments. Second, the P-Δ, (called the big P-delta), where Δ is related to member end displacements and should be considered in the numerical formulation to accurately model the second order structural response.

In a previous study [8], the authors formulated a displacement-based beam element for large deformations of plastic plane frames. The effect of axial force was included in small deflection theories; and the element was formulated in a body-attached coordinate to separate between rigid body and deformational rotations.

Another study presented a two-dimensional displacement-based and generalized mixed variational finite element that can be used to model arbitrary large displacements and rotations with small strains [9]. The

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research in [4] aimed to develop a three-dimensional force-based fibre beam element that considers inelastic large displacements. The algorithm is as a generalization of the state determination procedure presented in [10] for linear geometry/nonlinear material analysis, and the procedure described in [3] for linear material/nonlinear geometry. However, the element was only used to investigate the performance of steel frames under static monotonic loads. The element state determination was implemented in the software packages FedeasLab and OpenSees.

The authors in [5] presented several beam column finite element formulations for full nonlinear distributed plasticity analysis of two-dimensional steel frame structures. For the displacement-based and the mixed elements, the second order effect was included in the corotational formulation. Another research work in [11] promoted a numerical model for non-linear large-displacement dynamic analysis of steel beam-columns. The model was utilized to investigate the behaviour of beam-column steel elements subjected to blast loading. The steel members were restrained at their ends by rotational and translational springs producing second order effects. Further, the study in [6] developed a 3D distributed plasticity beam element using mixed formulations for composite circular concrete-filled steel tubes. The formulation considered large displacements and rotations using a corotational frame transformation.

Recently, a new study used a large displacement corotational formulation to analyse planar functionally graded sandwich beams [12]. The beams were composed of a metallic steel core and two top and bottom ceramic faces. The study highlighted the importance of considering the effect of plastic deformation in large displacement analysis.

In this paper, two planar fibre beam elements are presented for the analysis of slender RC members under cyclic loads. The first one uses a displacement-based technique to calculate the stiffness and the resisting forces of members. In this method, the equilibrium is satisfied in a weighted integral sense. For this technique, the use of a fine mesh is essential in plastic zones in order to represent precisely the curvature and strain distributions. The second element uses a mixed-based technique, where both displacements and internal forces are interpolated independently and the equilibrium is satisfied in a section by section basis. The mixed method requires less number of finite elements to simulate structural responses; however its state determination algorithm is much more complex.

The proposed elements are based on the work by [13,5]; to incorporate second order effects into displacement and mixed-based elements. Unlike the element of [5], which was used to analyse simple steel members under static monotonic loads only, the proposed elements developed herein are able to model the complex behaviour of normal and high-performance reinforced concrete as well as steel members under both monotonic, and severe cyclic loads. They can also monitor the behaviour of the structure at the element, section and fibre level. Further, the state determination process of the elements is modified for improved numerical efficiency.

The newly developed elements will be used to analyse RC members under different static and dynamic loading conditions. They take into account the geometric nonlinearity and benefit from sophisticated material models that can accurately simulate the nonlinear behaviour of concrete and steel materials, which will help in studying local effects in details. The elements are implemented in the research-oriented finite element analysis program FEAP developed by Taylor [14].

2. Transformation between corotational and global systems

The two elements formulated in this chapter follow Navier's three principles of mechanics: The stress equilibrium, the strain compatibility and the constitutive relationships of steel and concrete. First the two elements are formulated in a corotational system where rigid body modes are removed and small strains but large displacements are assumed. For the present formulation, the axial force is constant and does not change along the element, while distributed loads are not considered in the current fibre beam element formulation. Only internal loads on the members are lumped at nodal points along the members, and are transformed to the end loaded members.

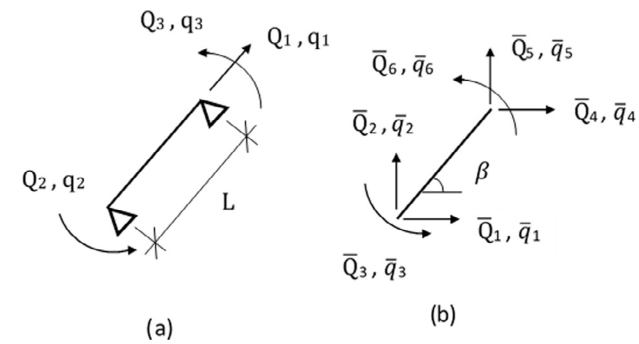

Fig. 1. Element forces and displacement degrees of freedom in: (a) corotational and (b) global system.

The matrix T_r links the element nodal forces in the global system with the element internal forces in the corotational system [5]:

$$\overline{Q} = T_r^T Q \tag{1}$$

$$T_r = \begin{bmatrix} -\frac{\sin\beta}{L} & \frac{\cos\beta}{L} & 1 & \frac{\sin\beta}{L} & -\frac{\cos\beta}{L} & 0 \\ -\frac{\sin\beta}{L} & \frac{\cos\beta}{L} & 0 & \frac{\sin\beta}{L} & -\frac{\cos\beta}{L} & 1 \\ -\cos\beta & -\sin\beta & 0 & \cos\beta & \sin\beta & 0 \end{bmatrix} \tag{2}$$

where $\overline{Q}$ and Q are the nodal forces in the global and corotational systems respectively, and are shown in Fig. 1), and β is the final angle of the deformed beam element:

$$\beta = \arctan\left(\frac{(y_2 + v_2)-(y_1 + v_1)}{(x_2 + u_2)-(x_1 + u_1)}\right) \tag{3}$$

where u is the end displacement in the horizontal direction and v is the end displacement in the vertical direction. Subscripts 1 and 2 refer to the element ends respectively.

In addition, the transformation matrix T_r is also used for the transformation of the displacements between the corotational and global system:

$$\delta\overline{q} = T_r^T \delta q \tag{4}$$

where $\overline{q}$ and q are the element end displacements in the global and corotational systems respectively.

Similarly, the stiffness matrix is transformed between the two systems using the same mapping matrix. However, an additional term K_G that includes the effects of element internal forces on the element stiffness must be included:

$$K_{elem(global)} = T_r^T K_{elem} T_r + K_G \tag{5}$$

where K_G is the well-established external geometric stiffness matrix.

3. Formulation of the displacement-based element

In the classical displacement-based method, the equilibrium is achieved only in a weighted integral sense. The displacements serve as primary variables and the principle of virtual displacements is implemented to obtain the solution.

The Green–Lagrange strain of the element reference axis in the natural frame that is derived from the displacement field can be defined as:

$$\hat{\varepsilon} = \frac{du}{dx} + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 + \frac{1}{2}\left(\frac{du}{dx}\right)^2 \tag{6}$$

where the transverse and axial displacements v and u are represented,