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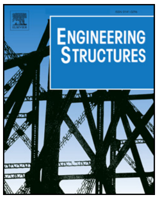

Modal analysis and dynamic response of a two adjacent single degree of freedom systems linked by spring-dashpot-inerter elements

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ABSTRACT

This paper illustrates the dynamics of a novel structural configuration represented by a two adjacent single-degree-of-freedom (2-ASDOF) systems coupled by a connection with spring-dashpot-inerter elements arranged in parallel. The connection is conservative if the link is realized only with spring and/or inerter, whereas becomes nonconservative when also the dashpot is added. The first part of the study concerns the system modal analysis: different linking schemes are considered in order to investigate the influence of each connection parameter on the coupled system modal properties. It is shown that, for conservative connection, modal properties are analytically derived because the eigenvalue problem admits closed-form solutions. For nonconservative connection, instead, the state-space description is adopted and the complex modal analysis is carried out (deriving pseudo-frequencies, pseudo-damping factors and complex modes). Admissible and not-admissible zones for the frequencies of the coupled system are individuated: frequencies are positioned on the left, on the right or within the uncoupled ones, depending on the values chosen for the connection parameters. Associated to each case, typical patterns for mode shapes are depicted. Peculiar behaviors of the system, advantageous for structural control purposes, emerge when the connection parameters assume specific values. The second part of the paper illustrates the system dynamics to very simple input conditions: the response to free vibration with initial conditions and to base harmonic motion is carried out. Based on the findings obtained from modal analysis, it is highlighted that it is possible to properly select the connection parameters in order to have: (i) the most rapid decay of the structural response in case of free vibrations and (ii) the minimum response amplification for both oscillators in case of harmonic motion.

Nomenclature

b	connection inertance
c	connection viscous damping
c_i	i-th SDOF system viscous damping coefficient
$e(t)$	applied input
g	acceleration of gravity
	element of row i and column h of damping matrix
j	imaginary unit
k	connection stiffness
k_i	i-th SDOF system stiffness
k_{ih}	element of row i and column h of stiffness matrix
m_i	i-th SDOF system mass
m_{ih}	element of row i and column h of mass matrix
$\boldsymbol{q}(t)$	non-dimensional kinematic parameter
$s_i,\ \bar{s}_i = a_i \pm jd_i$	i-th eigenvalue
$\boldsymbol{u}(t)$	relative displacement vector
$u_i(t)$	i-th SDOF system relative displacement
$\ddot{u}_G(t)$	base acceleration
$\boldsymbol{y}(t)$	output vector
$\boldsymbol{z}(t)$	state vector
A	harmonic excitation amplitude
$\boldsymbol{A}$	state (or system) matrix
$\boldsymbol{B}$	input influence matrix
$\boldsymbol{C}$	influence output matrix
$\boldsymbol{D}$	feed-through matrix
IE	inerter element
$\boldsymbol{K}$	stiffness matrix
$\boldsymbol{L}$	damping matrix
$\boldsymbol{M}$ and $\widetilde{\boldsymbol{M}}$	mass matrices
OF	objective function
RC	rigid connection
SE	spring element
SIE	spring-inerter element

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β	inerter mass ratio
β_{min}	minimum defective system inerter mass ratio
γ_i	i-th natural or pseudo-natural frequency
γ_{RC}	rigid connection frequency
η_i	i-th mode pseudo-damping ratio
λ	connection stiffness ratio
λ^* (i.e. $(\beta/\lambda)^*$)	defective system connection stiffness ratio
μ	2-DOF system mass ratio
ν	2-DOF system frequency ratio
ξ	connection damping ratio
ξ_i	i-th SDOF system damping factor
ξ^*	defective system connection damping ratio
ρ	forcing frequency ratio
$\boldsymbol{\tau}$	influence vector
υ_i	i-th mode ratio
$\boldsymbol{\phi_i}$	i-th mode
$\boldsymbol{\psi}_i$, $\overline{\boldsymbol{\psi}}_i = \boldsymbol{\phi}_i \pm j\boldsymbol{\chi}_i$	i-th eigenvector
ω_f	harmonic excitation circular frequency

Where subscripts $i = 1, 2$ and $h = 1, 2$ and a dot over a symbol represents differentiation respect to time.

1. Introduction

Scientific studies proved that an effective method to mitigate the dynamic response of two adjacent structures is to connect them with special control devices able to provide suitable reactive forces when relative motion is imposed to their terminals. A simple model to study the dynamic response of two adjacent structures is represented by a pair of linear elastic oscillators. The kind of connection chosen to link the two oscillators alters the dynamic behavior of the entire system. Coupling elements typically studied are springs, dampers, linear or nonlinear, passive or active, configured in series or parallel. Meirovitch [1] studied the free vibrations of two simple oscillators linked by a linear spring and obtained the closed-form expression of natural frequencies and modal vectors. Zhang and Xu [2] determined dynamic characteristics of adjacent buildings linked by viscoelastic dampers represented by the Voigt model (spring and dashpot arranged in parallel). Zhang [3] presented an accurate and effective procedure for determining dynamic characteristics and seismic response of adjacent buildings linked by a fluid damper represented by a Maxwell model (spring and dashpot arranged in series). A series of studies has also been devoted to dissipative linking elements realized through hysteretic dampers [4,5], friction dampers [6] or semi-active devices [7,8].

Recently, many authors have been studying systems for vibration control based on a new device called inerter, initially proposed by Smith in the 2000s [9] and used for the first time with the aim of increasing the performance of the vehicle suspensions [10]. Ideally, this new device is able to generate an inertia force proportional to the acceleration difference between its two terminals and to a constant called inertance that has the physical dimensions of a mass. The further feature of the device lies in the fact that the inertial mass (or inertance) can also be two orders of magnitude larger than the gravitational mass of the device. From a technological point of view, the inerter can be mechanical (pinion and rack or screw and balls [11]) and hydraulic [12]. Born as passive devices, semi-active inerters have been also recently developed, e.g. [13,14]. Specifically, a physical realization of a novel semiactive-inerter-based adaptive tuned vibration absorber is presented in [15].

The potential of the inerter makes it particularly interesting in civil engineering applications and, in particular, for the realization of a “light” Tuned Mass Damper (TMD); in fact, in the very last years, new TMD inerter-based systems for structural vibration suppression have been proposed.

In 2012, Ikago et al. [16] proposed the Tuned Viscous Mass Damper (TVMD) applied to a single-degree-of-freedom (SDOF) structure subjected to harmonic excitation, deriving a closed-form solution for optimum seismic control design. In the 2013, Garrido et al. [17] proposed the Rotational Inertia Double-Tuned Mass Damper (RIDTMD) an SDOF structure under harmonic excitation and obtained numerically optimum design parameters. In the 2014, Lazar et al. [18] proposed the Tuned Inerter Damper (TID) inside a multi-storey building. Hu et al. [19] evaluated the performance of an inerter-based dynamic vibration absorber with a parallel arrangement of a spring and an inerter-based mechanical network. Finally, the Tuned Mass Damper Inerter (TMDI), proposed by Marian and Giaralis [20] and Pietrosanti et al. [21], has been demonstrated to be more effective and robust with respect to conventional and non-conventional TMD systems [22,23]. Chen et al. [24] studied the influence of inerter on natural frequencies of vibration systems considering: a SDOF system, a 2-DOF system obtained by coupling two SDOF in series and, finally, a multi-DOF (MDOF) system.

This paper presents the dynamics of a 2-DOF system consisting of two adjacent SDOF (2-ASDOF) structures coupled by a connection with spring-dashpot-inerter elements arranged in parallel and proposed by the same authors in [25]. The main novelty lies in the utilization of an inerter as linking element between adjacent structures. This study represents the first part of a wider research project, which has, as final objective, the purpose to explore the use of an inerter - in conjunction with other rheological elements - for the mitigation of the dynamic response of adjacent structures. In the first part of the paper, modal analysis of such a coupled system is presented, considering a conservative (spring element, inerter element, spring-inerter elements) and nonconservative (spring-damper-inerter elements) connection. The objective is to investigate how the connection parameters influence the modal properties of the coupled system. In case of conservative connection, by way of a linear formulation of the problem and through a very simple model, full information about the dynamic behavior of the system with the derivation of closed-form expressions for the natural frequencies and mode shapes are obtained. In case of nonconservative connection, the state-space description is required and the modal analysis implies complex solutions for the eigenvalues and the eigenvectors: as a result, by solving the algebraic eigenvalue problem, pseudo-frequencies, pseudo-damping factors and complex modes are derived. In the second part of the paper, the system is subject to simple input motion conditions: the response to free vibration with initial conditions and to base harmonic motion is investigated. Even if the motion conditions are straightforward, useful information on the basic dynamics of the coupled system with conservative and nonconservative connection are given. From dynamic analysis, the main findings obtained by modal analysis are confirmed and a way to select the connection parameters, in order to reach minimum response amplification for both oscillators, is suggested.

The organization of the paper is as follows. Section 2 describes the position of the problem and the equations of motion in the most general case of a viscously damped 2-DOF system linked by a spring-dashpot-inerter adjusted in parallel. Section 3 describes the eigenvalue problem for the cases of conservative and nonconservative connection. Sections 4 and 5 discuss the results of modal analysis for the cases of conservative and nonconservative connection respectively. Section 6 discusses the results of the free vibrations in the cases of conservative and nonconservative connection. Section 7 discusses the results of base harmonic motion in the cases of conservative and nonconservative connection. Conclusions with major findings of the work are drawn in Section 8.

2. Position of the problem and equations of motion

Consider a viscously damped 2-DOF system, represented by 2-ASDOF systems linked by spring-dashpot-inerter elements (SDIE)