Engineering Structures 174 (2018) 851–860

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Dual cohesive elements for 3D modelling of synthetic fibre-reinforced concrete

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ARTICLE INFO

Keywords:
Fibre reinforced concrete
Interface cohesive elements
FEA
Discrete fibre-modelling
Dual cohesive model

ABSTRACT

In this paper, we introduce a novel computational approach that describes dual scale discontinuities where the fibres and fractures interfere and influence the overall behaviour of a structure. A mathematical formulation is proposed in alignment with the finite element method to take into account the contribution of each component as a discrete entity. The formulation is implemented numerically through the Fortran User Element subroutine UEL of the general purpose software Abaqus. This novel approach that we call "dual cohesive element" is suitable for synthetic macro fibre-reinforced concrete. It can represent the overall structural behaviour with a coarse discretisation which makes it particularly fast and efficient. Once a certain fibre distribution across the crack is postulated, single fibre pull-out test results can be used to simulate the load-displacement performance of the structure. This numerical method is validated experimentally and compared with single fibre pull-out tests and three point bending beam tests which proved its accuracy in predicting crack opening in Mode I. A mesh refinement study is also conducted and the structural response obtained from the numerical simulation show mesh independent results. There is good agreement between experimental and simulated results even with a coarse mesh representation. The results highlight the intimate relationship between single fibre pull-out test and the behaviour of fibre reinforced concrete with multiple fibres.

1. Introduction

Fibre reinforcement is a common way of enhancing the properties of a host material (often referred to as matrix) by means of embedded fibres hence forming a composite material. In concrete, which is well known to manifest a brittle behaviour, fibres have the ability to increase the post-peak performance by ensuring residual strength, thereby leading to higher safety. The superiority of fibre-reinforced concrete over conventional concrete in terms of ductility is well established, however the way fibres are dispersed into concrete, affects the overall performance of the composite materials and develops a higher scatter of results. Nevertheless, the influence of the orientation and position of fibres in bridging the cracks has been, in fact, the object of numerous studies and is a well known phenomenon [1–4].

Several attempts have been made to model the performance of fibre-reinforced concrete and there is a plethora of tools available to predict the responses of such material under various loading conditions. Most approaches are based on the cohesive crack model developed by Hillerborg [5] which requires knowledge of a $\sigma{-}w$ or stress-CMOD (Crack Mouth Opening Displacement) response. In this context the effect of the fibre is spread throughout the crack and treats fibre-reinforced concrete as a homogeneous material [6–8]. Since the fibre distribution is not taken directly into consideration, the $\sigma{-}w$ response is often obtained experimentally based on random sampling with possible poor statistical representation of large civil structures.

In recent years, another approach based on discrete modelling of fibres has proven to be a valuable tool since the contribution of each single fibre can be taken into account directly without the need to use inverse techniques to extrapolate the $\sigma{-}w$ relationship. The distribution of fibres within the concrete matrix, however, must be postulated. In [9], for example, the effect of discrete fibres embedded in concrete is modelled from the fibre pull-out response by assuming that the forces take place at the fibre ends. The effect of these forces is then spread in an area of influence located at the vicinity of the ends of the fibres to avoid stress concentration as a consequence of mesh refinement. The solid elements are linear elastic until certain conditions are met to follow an exponential softening law.

Cunha et al. [10] propose a model that relies on the fibre pull-out response but the effect of fibres is described using embedded 3D elements. These embedded 3D elements representing the fibres are also independent of the bulk mesh but require an inverse mapping technique to find out the coordinates of the points where the fibres are

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https://doi.org/10.1016/j.engstruct.2018.07.101
Received 16 March 2018; Received in revised form 22 June 2018; Accepted 30 July 2018

Nomenclature

(ξ, η)	intrinsic coordinates
(ξ_f, η_f)	intrinsic coordinates corresponding to (x_f, y_f, z_f) in the 3D space
(x, y, z)	Cartesian coordinates defining the 3D Euclidean space, also called extrinsic coordinates
(x_f, y_f, z_f)	3D coordinates of a point where the fibre is intersecting the dual cohesive element and where the force $\{\boldsymbol{P_f}\}$ is applied
$[\boldsymbol{k_f}]$	matrix containing the "stiffness" terms that relate the crack displacement to the bridging force of the fibre
$[\Delta\boldsymbol{G}]$	Green's strain tensor
$\{\boldsymbol{u}\}$	displacement vector containing the separation between the two faces of the dual cohesive element
$\boldsymbol{X}$	three-dimensional Euclidean space
δ	Delta Dirac function
Γ_c	area of the domain in which $\{\boldsymbol{T}\}_{\text{coh}}$ is defined
Γ_e	area of the domain in which $\{\boldsymbol{T}\}_{\text{ext}}$ is defined
Γ_n	area of the domain in which $\{\boldsymbol{R}\}$ is defined
nf	total number of fibres intersecting the cohesive element
$[\boldsymbol{B}]$	matrix containing the shape functions
$[\boldsymbol{D}]$	constitutive matrix that relates the separation of the two faces of the cohesive element to the cohesive stresses
$[\boldsymbol{J}]$	matrix containing the terms of the transformation between extrinsic and intrinsic coordinates
$[\boldsymbol{K}_{\text{el}}]$	dual cohesive element's stiffness matrix
$[\boldsymbol{K}_{\text{el}}]_\sigma$	component of the dual cohesive element's stiffness matrix due to the cohesive stresses
$[\boldsymbol{K}_{\text{el}}]_f$	component of the dual cohesive element's stiffness matrix due to the bridging action of the fibre
$[\boldsymbol{R}]$	rotation matrix of the dual cohesive element
$[\boldsymbol{S}]$	second Piola-Kirchoff tensor of the internal stresses
$\{\boldsymbol{\sigma}\}$	cohesive stress component acting on the dual cohesive element
$\{\boldsymbol{d}\}$	displacement vector at the nodal point of the dual cohesive element
$\{\boldsymbol{d}\}_{\text{vir}}$	vector containing the virtual displacement at the nodal points
$\{\boldsymbol{F_e}\}$	vector containing the magnitude of the external forces acting at the nodal point of the dual cohesive element
$\{\boldsymbol{F_e}\}_f$	vector containing the magnitude of the external forces acting at the nodal point of the dual cohesive element due to the bridging action of the fibre
$\{\boldsymbol{F_e}\}_\sigma$	vector containing the magnitude of the external forces acting at the nodal point of the dual cohesive element due to the cohesive stresses
$\{\boldsymbol{P_f}\}$	vector containing the magnitude of the bridging forces due to a fibre
$\{\boldsymbol{R}\}$	vector containing the magnitude of the reactions
$\{\boldsymbol{T}\}_{\text{coh}}$	vector containing the magnitude of the cohesive forces acting on the discontinuity (crack)
$\{\boldsymbol{T}\}_{\text{ext}}$	vector containing the magnitude of the external imposed forces
$\{\boldsymbol{u}\}_{\text{vir}}$	virtual displacement between the two faces of the dual cohesive element
Ω	3D domain of the volume
$\sigma - w$	stress-crack opening relationship
$\mathrm{d}A$	area element in the Euclidean space
$\mathrm{d}V$	volume element in the Euclidean space
$\{\Delta\boldsymbol{u}_c\}$	vector containing the magnitude of the separation of the discontinuity
$\{\Delta\boldsymbol{u}_e\}$	vector of the virtual displacements within the domain Γ_e
$\{\Delta\boldsymbol{u}_n\}$	vector of the virtual displacements within the domain Γ_n
A_e	domain of the dual cohesive element
f_t	ultimate tensile stress
$G_f^{\text{II}}, G_f^{\text{III}}$	fracture energy in Mode II and Mode III
G_f^{I}	fracture energy in Mode I
J	Jacobian of the transformation between extrinsic and intrinsic coordinates
k_{in}	initial stiffness coefficient that populates the $[\boldsymbol{D}]$ matrix also called penalty factor
$N_{1,\ldots,4}$	shape functions that populates the matrix $[\boldsymbol{B}]$
W_{ext}	external work as defined in the Principle of Virtual Work
$W_{\text{ext}}^{\text{el}}$	external work as defined in the Principle of Virtual Work limited to a dual cohesive element
W_{int}	internal work as defined in the Principle of Virtual Work
$W_{\text{int}}^{\text{el}}$	internal work as defined in the Principle of Virtual Work limited to a dual cohesive element
CMOD	crack mouth opening displacement
FRC	fibre reinforced concrete
UEL	user-defined Element subroutine in Abaqus

intersecting the solid elements. The 3D embedded elements are integrated over the length of the fibre which requires knowledge of fibre elastic parameters such as the Young and Shear moduli. These moduli may not be easily measured through a conventional pull-out test and assume a certain stress distribution over the length of the fibres. The stiffness matrix for such elements containing the effect of one or more fibres is updated by superposing the effect of each fibre. The damage within concrete is modelled with the fixed smeared crack method. These authors employ an algorithm to generate the positions and orientations of fibres that also takes into account the physical boundaries of the solid (wall effect) based on Cunha's approach [11].

To overcome some of the issues related to modelling cracks with a smeared crack model used in [9,10], Octavio et al. [12] adopted the Conforming Generalised Strong Discontinuity Approach that reads the displacement field as made up from a contribution of the deformation of the material and another caused by the discontinuity. The jump in displacement at the fibre level is interpolated from that of the corresponding enriched elements and it is related to the strain. To calculate the contribution of fibres the model requires the stress-strain relationship of the fibre.

Furthermore, efforts have been made to model fibre-reinforced composites without employing FEA (Finite Element Analysis). In [13] for example the authors represent the behaviour of fibre-reinforced materials with an event based lattice model while [14] handle the problem with a meshless peridynamic analysis. Both studies make use a of discrete characterisation of fibres. Finally, there is also a number of studies that either directly use, or idealise, the pull-out response to model FRC analytically [15–17]. These models however can only be applied to certain structures as they use theories of solid mechanics. On this regard numerical Finite Element analysis is more flexible.

The majority of studies that focus on modelling the contribution of fibres as a single entity require the user to input a stress-displacement or stress-deformation response of the fibre which often requires inverse analysis to gather this information.

To the knowledge of the authors, at the moment there is no formulation of a numerical approach that features force-displacement at the fibre level without the assumptions on the stress distribution and, at the same time, can model the physical separation that happens during the onset of cracking.

In this study, we present a methodology that can be implemented in an established framework like Finite Element analysis, models the fibres and crack as two distinct and discrete entities and can benefit from the fact that all the input parameters can be obtained from a simple test: the single fibre pull-out test.