Engineering Structures 173 (2018) 853–869

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

On the geometry design of the stiffened pipe structure: A finite element model

Ming Fang[a,b], Tianyu Li[c,*]

[a] State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China
[b] University of Chinese Academy of Sciences, Beijing 100049, China
[c] Aerospace and Ocean Engineering Department, Virginia Tech, Blacksburg, VA, USA

ARTICLE INFO

Keywords:
Stiffened pipe
Variable thickness pipe element
Smooth configuration
Consistent Orthogonal Basis Function Space
Buckling analysis
ANSYS

ABSTRACT

The stiffened pipe structure is analyzed by using a pipe element with variable cross-section. The pipe element uses a much less number of DOF (degree-of-freedom) and it has exactly smooth configuration and high order solution of displacement. The Consistent Orthogonal Basis Function Space is applied to define the pipe's displacement basis functions. The nonlinear finite element implementation details are also presented for the completeness of the method.

The stiffened pipe collapse problem and the buckling propagation problem are both solved. The numerical results are compared and verified with ANSYS. It is observed that the stiffness can be largely increased by using stiffeners on pipes. The number of DOF is also compared between the proposed method and ANSYS. The post buckling profile of the stiffened pipe is obtained. The maximum and minimum buckling propagation pressure are calculated. The design based on buckling collapse and buckling propagation are discussed.

1. Introduction

Pipe structures have been applied in many engineering fields. Relative research work is reviewed here. The variable thickness pipe element and the static/dynamical buckling analysis are developed in [1,2]. In these two papers, the problems of pipe buckling cross-over and dynamical buckling propagation are solved, respectively. In [1], the proposed pipe element is used to simulate the pipeline with integral arrestors. The critical cross-over pressure is calculated. In [2], the pipe element is generalized to dynamical case. For all these problems, nonlinear finite element for the large deformation and finite strain analysis are considered. For the nonlinear finite element background, it is referred to the book [3]. In this book, the nonlinear FEM (finite element method) equations are comprehensively derived. The plastic constitutive code is developed based on the algorithm in [4], from which the return-mapping algorithm is used.

In [5], the research history of pipe finite element is presented. In [6,7], an elbow pipe finite element is developed for linear and nonlinear analysis. The axial line of the pipe can be a curve. It could be used to the connection of two pipes. Comparing to this paper's method, the limitation of [6,7] are:

(1) The cross-section of pipes could only be a perfect circle, while in this paper, the cross-section could be any shape.
(2) Ref. [6] is a linear pipe element, and [7] only considers few geometrical nonlinearity.
(3) Refs. [6,7] only considers bending behavior, while this paper includes all the deformation (bending, membrane, shear, extension, compression etc).

The beam element with a deformable cross-section is developed in [8]. In [8], the pipe element is based on beam theory in axial direction and a 3-D warping displacement based on solid elements. It is also able to represent any shape of cross-section since a tradition 2-D mesh is used for cross-sections. However, comparing to this paper, it must use a much more number of degree-of-freedom for each cross-sections.

In [9], the bending pipe element is developed. In this paper, it is based on Von Karman ovalisation. However, only thin-walled pipe is analyzable. Linear elastic analysis is only considered in [9]. The ANSYS analysis of pipe under pressure is given in [10]. In [11], the ovality and thickness are studied for the analysis of buckling collapse based on traditional FEM package. The stiffened plate under compression is studied in [12]. The super element is used and nonlinear strain due to transverse deflection is considered. A linear Timoshenko beam theory is used, such that only thin-walled stiffener is analyzable. In [13], the ANSYS is applied to analyze the problem of cylinder shell buckling. The

* Corresponding author.
E-mail addresses: mingfang@siom.ac.cn (M. Fang), lty1990@vt.edu (T. Li).

https://doi.org/10.1016/j.engstruct.2018.06.075
Received 4 June 2017; Received in revised form 9 May 2018; Accepted 20 June 2018

vibration of a variable thickness shell is analyzed in [14]. Donnel-Mushtari conical shell theory is used and the Ritz method is applied to calculate the strain energy. In [15], the elbow pipe under internal pressure is studied. The key contribution of [15] is the elaplastic model for the elbow pipe. An analytical solution of the limit load is obtained. Linear problem is considered. In [16], the bending buckling analysis of straight pipe and curved pipe is presented. In [17], the composite structure with curved stiffeners is focused and the optimization based on buckling analysis is discussed. In [18], the stiffened shell buckling sensitivity to geometrical imperfection is studied. In [19,20], the pipe element is generalized into curved pipe and laminated pipe. In [19], the axial line of pipe can be any 3-D space curve. Also, the cross-section is still arbitrary and variable along the curved pipe's axial line. In [20], the laminated pipe element is developed. The cross-section can have many layers with different materials and the interface stress can be calculated directly. In [21], the buckling propagation of pipe-in-pipe is studied. The pipe-in-pipe system is applied to mitigate the vibration in [22], where a finite element analysis and a simplified TMD model are presented. In [23], a novel contact/impact algorithm is developed for the pipe-in-pipe structure. The location of contact/impact between the inner pipe and the outer pipe can be determined directly based on the deformed pipe's arc length.

In this paper, the stiffened pipe structure is analyzed by using a pipe element with arbitrary variable cross-section. The method can be used to analyze any pipes with any shape of cross-section and the cross-section can be variable along the pipeline. In this case, it has wide applications. The number of DOF is also much less comparing to traditional finite element method (FEM). The configuration of the pipe structure can be also represented exactly and accurately.

By using this method, the stiffened pipe buckling problem is studied in this paper. Comparing to the previous work in this field, this paper firstly studies the relation between geometrical parameters of stiffened pipes (ribs thickness, ribs distance, etc.) and strength/stability of the stiffened pipe structure systematically.

The outline of the paper is given. In Section 2, the kinematics of the stiffened pipe is discussed. In Section 3, the Consistent Orthogonal Basis Function Space is presented. In Section 4, the nonlinear finite element implementation details are given. In Section 5, the numerical tests are presented. In Section 6, the conclusion is given.

2. Kinematics

In this section, the kinematics of the variable cross-section pipe is established. This pipe element is used in this paper to analyze the stiffened pipe problem. The key idea of pipe element is to set a mapping between a perfect cylinder space to a torsional body space with arbitrary variable cross-section. Then, the natural coordinate system is defined in the perfect cylinder space, and the material coordinates for the torsional body with arbitrary variable cross-section can be represented.

The natural coordinate system ξ_1, ξ_2, ξ_3 is shown in Fig. 1 [1,2].

In Fig. 1, ρ, θ, z are the commonly-used cylindrical coordinates, and ξ_1, ξ_2, ξ_3 are the natural coordinates, as defined in (Eq. (1a,b,c)).

$$\xi_1 = z/L$$
$$\xi_2 = 2\theta/\pi$$
$$\xi_3 = (\rho - R)/t_w \qquad \text{(1a,b,c)}$$

where L is the total length of the pipe in the axial direction. O' is the origin of the cylindrical coordinate system ρ, θ, z as seen in Fig. 1, $O'O''$ is the *z-axis*, O is the origin point of the ξ_1, ξ_2, ξ_3 coordinate system, OO' is the distance between the origin point of the cylindrical coordinate system ρ, θ, z and the inner surface of the pipe. PO' is perpendicular to OO', and P belongs to the outer surface of pipe.

It should be noted that the wall thickness t_w can be a function of ξ_1, ξ_2. O' can be placed at any point within the body of the structure, but the formulation of $OO' = R(\xi_1, \xi_2)$ will be adjusted correspondingly. The definition of $R = R(\xi_1, \xi_2)$ is the distance between the origin point O' and the pipe's internal surface, as shown in Fig. 2. The function $R = R(\xi_1, \xi_2)$ can be dependent on axial direction $\xi_1 = z/L$ and circumferential direction $\xi_2 = 2\theta/\pi$. Similarly, the distance between the origin point O' and the outer surface of the pipe can be also a function of (ξ_1, ξ_2), which is denoted as $R' = R'(\xi_1, \xi_2)$, as seen in Fig. 2. Thus, the definition of $t_w = t_w(\xi_1, \xi_2)$ is $t_w = R' - R$, which is also a function of (ξ_1, ξ_2).

The displacement expression of the pipe element is given in (Eq. (2a,b,c)). The element displacement is defined based on the natural coordinates ξ_1, ξ_2, ξ_3 as:

$$u_1 = R_0 f_{u_1}^{\xi_1} f_{u_1}^{\xi_2} f_{u_1}^{\xi_3} \sum_{m,n,k=1}^{m_0,n_0,k_0} A_{mnk}\phi_{u_1}^{m}(\xi_1)\psi_{u_1}^{n}(\xi_2)\varphi_{u_1}^{k}(\xi_3)$$
$$u_2 = R_0 f_{u_2}^{\xi_1} f_{u_2}^{\xi_2} f_{u_2}^{\xi_3} \sum_{m,n,k=1}^{m_0,n_0,k_0} B_{mnk}\phi_{u_2}^{m}(\xi_1)\psi_{u_2}^{n}(\xi_2)\varphi_{u_2}^{k}(\xi_3)$$
$$u_3 = R_0 f_{u_3}^{\xi_1} f_{u_3}^{\xi_2} f_{u_3}^{\xi_3} \sum_{m,n,k=1}^{m_0,n_0,k_0} C_{mnk}\phi_{u_3}^{m}(\xi_1)\psi_{u_3}^{n}(\xi_2)\varphi_{u_3}^{k}(\xi_3) \qquad \text{(2a,b,c)}$$

where u_1, u_2, u_3 are denoted as the axial, circumferential and radial displacement, respectively; ξ_1, ξ_2, ξ_3 are the dimensionless cylindrical coordinates defined in Fig. 1 and Eq. (1); m_0, n_0, k_0 are the displacement order for ξ_1, ξ_2, ξ_3, respectively; R_O is a referential radius that is used to non-dimensionalize the degree-of-freedom; ϕ_{u_i}, $i = 1, 2, 3$ is the Lagrangian interpolation function about axial natural coordinate ξ_1 for u_i, $i = 1, 2, 3$; $\psi_{u_i}(\xi_2)$, $\varphi_{u_i}(\xi_3)$ are the displacement basis function about circumferential coordinate ξ_2 and radial coordinate ξ_3, respectively, for u_i, the specific formulation of $\psi_{u_i}(\xi_2)$, $\varphi_{u_i}(\xi_3)$ will be discussed later in Section 3; A_{mnk}, B_{mnk}, C_{mnk} are the undetermined coefficients of the displacement expression, which is the DOF (degree-of-freedom); $f_{ui}^{\xi_j}$ is the displacement boundary condition function, which is used to define the displacement boundary condition about the natural coordinate ξ_j for u_i. If there is no displacement boundary condition about the circumferential coordinate, it will be defined as 1.

In the axial direction, since the Lagrangian interpolation function is used, the displacement compatible condition must be satisfied. For the two adjacent pipe elements denoted as element i and element $i+1$, the displacement continuity condition is satisfied by reinforcing (Eq. (3a,b,c)).

$$A_{m_0nk}^{i} = A_{1nk}^{i+1}$$
$$B_{m_0nk}^{i} = B_{1nk}^{i+1}$$
$$C_{m_0nk}^{i} = C_{1nk}^{i+1} \qquad \text{(3a,b,c)}$$

Eq. (3a,b,c) means that A_{mnk}, B_{mnk}, C_{mnk} of element i for which $m = m_0$ must be equal to A_{mnk}, B_{mnk}, C_{mnk} of element $i+1$ for which $m = 1$ individually.

For element i, A_{mnk}, B_{mnk}, C_{mnk} $(m = m_0)$ refer to the degrees at the mth Lagrangian interpolation point in the axial direction. For element $i+1$, A_{mnk}, B_{mnk}, C_{mnk} $(m = 1)$ refer to the degrees at the first Lagrangian interpolation point in the axial direction. Once Eq. (3ac) are satisfied, the compatibility condition between adjacent pipe elements will be satisfied. For the radial and circumferential direction, high order orthogonal polynomials will be used to define the displacement globally. So there is no need to address compatibility problem.

For the ith element, the configuration of the pipe element can be formulated as:

$$^{0}r = \begin{bmatrix} ^{0}x_1 \\ ^{0}x_2 \\ ^{0}x_3 \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{i-1} L^j + L^i\xi_1 \\ (R + t_w^i(\xi_1, \xi_2)\xi_3)\cos\left(\frac{\pi}{2}\xi_2\right) \\ (R + t_w^i(\xi_1, \xi_2)\xi_3)\sin\left(\frac{\pi}{2}\xi_2\right) \end{bmatrix} \qquad (4)$$