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Multiple analytical mode decompositions for nonlinear system identification from forced vibration

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ABSTRACT

In this study, multiple analytical mode decompositions (M-AMD) are proposed to identify the parameters of nonlinear structures from forced vibration. For the time-varying damping (or stiffness) coefficient of a weakly-to-moderately nonlinear system, the slow-varying part is first estimated from the system responses and their Hilbert transforms, which is corrected with an adaptive low-pass filter referred to as analytical mode decomposition (AMD). The fast-varying part can then be identified from the responses together with the estimated slow-varying part, which is again corrected with the AMD. The computational efficiency and accuracy of the proposed M-AMD are demonstrated with a Duffing oscillator subjected to harmonic loading. The errors in estimation of all model parameters are less than 3% from uncontaminated displacement responses, which is more accurate compared with the results from Hilbert spectral analysis. Changes of the fast-varying stiffness part have been taken into account with high accuracy. The M-AMD algorithm is then validated with a ¼-scale, 3-story building with one piezoelectric friction damper under earthquake excitations. The parameters of such a semi-active damper are identified with less than 1% error on average.

1. Introduction

Due to global warming, extreme events such as earthquakes, hurricanes, tornados, and floods have occurred more frequently and more severely in recent years. Civil engineering structures experience nonlinear deformation more often than ever before, leading to concrete cracks, steel yielding, and member buckling. Moreover, nonlinearity exists in supplemental energy dissipation devices, such as piezoelectric friction dampers (PFD) [1], even when the main structures under investigation remain in elastic range under these extreme events.

Time-frequency representation of a complete signal has been widely used in the identification of time-varying systems, including short-time Fourier transform [2], Wigner-Ville distribution [3], and wavelet transform [4]. It often results in a smeared or blurred scalogram in time-frequency domain since its limited resolution, constrained by the Heisenberg uncertainty principle in the case of wavelet transform, skews instantaneous or fast-varying components around a constant or slow-varying frequency ridgeline. Therefore, the so-called instantaneous frequency obtained often does not represent the local feature of the systems in time.

Hilbert transform has received increasing attention in the field of system identification and damage detection after the introduction of the well-known Hilbert-Huang transform (HHT) [5]. Since then, HHT has been applied to the detection of faults in mechanical systems [6,7], the system identification and damage detection of structures [8–10], and the analysis of earthquake records [11,12]. The essential element in the HHT is the formulation of empirical mode decomposition (EMD). In simple words, by decomposing a signal into many structured components, the frequencies of all vibration modes can be derived from the signal components, each giving rise to the frequency of a single mode. In engineering applications, however, EMD has mode mixing problems in mechanical and structural systems with closely-spaced frequencies. To ensure a proper separation of the frequency bandwidths of intrinsic modes, minimum cutoff frequency criteria were established in the EMD sifting [13], since subjective intervention to the EMD process may distort these mode functions. Wavelet packet transform was also used to decompose a signal into a set of narrowband components for the improvement of EMD analysis [14]. In addition, a multi-degree-of-freedom (MDOF) system was identified with the EMD enhanced by band-pass filter, which acts as a pre-processing tool for the measured free vibration time histories [8,9]. EMD was enhanced with the waves' beating phenomena to reveal the time-varying properties of the system embedded in the measured narrowband signals [15]. By injecting a known time function into the original signal, EMD can successfully

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decompose the small intermittent fluctuation around a large ocean wave [16]. Singular value decomposition (SVD) together with band-pass filtering techniques was also used to serve EMD for the extraction of the parameters with closely-spaced modes [17].

Hilbert transform was also utilized to extract the instantaneous modal parameter of a nonlinear system with FREEVIB and FORCEVIB algorithms [18–22]. However, the frequency and damping coefficients derived from the envelope and instantaneous frequency of the weakly-to-moderately nonlinear system did not take full considerations of the actual frequency variation over time. Hilbert vibration decomposition (HVD) was also developed to obtain the envelope of instantaneous frequency, which partially accounted for high harmonics [21]. HVD faced a challenge with noise-contaminated low amplitude harmonics [23], and was more applicable to deal with the free vibration of a single-degree-of-freedom (SDOF) system due to complexity in mathematical formulation. As the space between two modal frequencies decreases, the challenge for a consistent and reliable identification of the properties of linear structures remains.

Overall, the mode mixing problem in system identification with Hilbert transform has not been solved until analytical mode decomposition (AMD) was proposed as a mathematically proven, adaptive data analysis method [24]. Like the EMD and HVD, the AMD allows the inclusion of both main and super harmonics.

For nonlinear system identification, several other techniques have been proposed in the past decades. A constant forgetting factor was introduced to the least-square estimation for the identification of time-varying parameters of structures, leading to better tracking of instantaneous features [25]. However, such a small forgetting factor was very sensitive to measurement noise. An adaptive tracking technique was also proposed to identify system parameters and their changes due to damage [26]. If more data points were present, the accuracy of the calculated parameter was improved at the expense of computational effort with this tracking technique. Moreover, based on the least-square estimation, another adaptive parametric identification algorithm was developed to identify the nonlinear hysteretic restoring forces in a simple system [27]. The method was then extended to more complex nonlinear hysteretic behaviors [28,29]. Although successfully applied to the identification of the time-varying parameters of SDOF systems [30,31], the HHT proved to be less effective in application to MDOF systems due to the invalid orthogonality assumption introduced between two intrinsic mode functions.

In this study, multiple analytical mode decompositions (M-AMD) are proposed and developed to identify the stiffness and damping coefficients of weakly-to-moderately nonlinear systems from forced vibration. The efficiency and accuracy of the proposed M-AMD are evaluated with a characteristic nonlinear Duffing oscillator with hardening stiffness under harmonic loads and a ¼-scale 3-story building model with frictional damping under seismic excitations. The stiffness and damping coefficients identified from the M-AMD are compared with their respective instantaneous parameters directly obtained from Hilbert transform of the system responses.

2. The proposed M-AMD method under forced vibration

Consider a weakly nonlinear structural system represented by a SDOF oscillator. Let the mass-normalized time-varying damping coefficient, stiffness coefficient and external force be $2h(t)$, $\omega^2(t)$ and $p(t)$, respectively. The equation of motion for the forced vibration of the nonlinear system can be expressed into:

$$\ddot{x}(t) + 2h(t)\dot{x}(t) + \omega^2(t)x(t) = p(t) \tag{1}$$

in which $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ represent the displacement, velocity, and acceleration of the nonlinear system. Let $2h(t) = 2h_s(t) + 2h_f(t)$ and $\omega^2(t) = \omega_s^2(t) + \omega_f^2(t)$, where the subscripts s and f indicate slow- and fast-varying components of the normalized damping (or stiffness) coefficient with its frequency lower and higher than the narrow frequency band of velocity (or displacement). Eq. (1) can then be rewritten as:

$$\ddot{x}(t) + 2h_s(t)\dot{x}(t) + 2h_f(t)\dot{x}(t) + \omega_s^2(t)x(t) + \omega_f^2(t)x(t) = p(t). \tag{2}$$

By performing the Bedrosian identity [32], the Hilbert transform of Eq. (2) gives:

$$\begin{aligned} &H[\ddot{x}(t)] + 2h_s(t)H[\dot{x}(t)] + H[2h_f(t)]\dot{x}(t) + \omega_s^2(t)H[x(t)] \\ &\quad + H[\omega_f^2(t)]x(t) = H[p(t)] \end{aligned} \tag{3}$$

where $H[\cdot]$ stands for the Hilbert transform of a function inside the bracket. If the fast-varying components, $2h_f(t)$ and $\omega_f^2(t)$, are neglected as assumed with the FORCEVIB method [19], Eqs. (1) and (2) become a pair of simplified equations with new parameters $2h_0$ and ω_0^2 as follows:

$$\begin{cases} \ddot{x} + 2h_0\dot{x} + \omega_0^2 x = p \\ H[\ddot{x}] + 2h_0 H[\dot{x}] + \omega_0^2 H[x] = H[p] \end{cases} \tag{4}$$

Note that Eq. (4) represents an abbreviated form of time functions without explicitly showing the time variable. When the displacement, velocity, and acceleration responses are known, the solution of Eq. (4) for $2h_0$ and ω_0^2 can be expressed into:

$$\begin{cases} \omega_0^2 = \dfrac{pH[\dot{x}] - H[p]\dot{x}}{xH[\dot{x}] - H[x]\dot{x}} - \dfrac{\ddot{x}H[\dot{x}] - H[\ddot{x}]\dot{x}}{xH[\dot{x}] - H[x]\dot{x}} = \omega_{0p}^2 + \omega_{0x}^2 \\ 2h_0 = \dfrac{pH[x] - H[p]x}{H[x]\dot{x} - xH[\dot{x}]} - \dfrac{\ddot{x}H[x] - H[\ddot{x}]x}{H[x]\dot{x} - xH[\dot{x}]} = 2h_{0p} + 2h_{0x} \end{cases} \tag{5}$$

They are referred to as the initial instantaneous stiffness and damping coefficients, respectively. The subscript p indicates the stiffness/damping coefficient part that is directly associated with the load function, while the subscript x indicates the coefficient from the system responses only.

2.1. Slow-varying components

When $\omega_f^2 = 0$ and $h_f = 0$, $\omega_s^2 = \omega_0^2$ and $h_s = h_0$ are true. Otherwise, ω_s^2 and h_s are associated with ω_f^2 and h_f based on Eqs. (2) and (3):

$$\begin{aligned} \omega_s^2 = &\frac{pH[\dot{x}]-H[p]\dot{x}}{xH[\dot{x}]-H[x]\dot{x}} - \frac{\ddot{x}H[\dot{x}]-H[\ddot{x}]\dot{x}}{xH[\dot{x}]-H[x]\dot{x}} - \frac{(2h_f H[\dot{x}]-H[2h_f]\dot{x})\dot{x}}{xH[\dot{x}]-H[x]\dot{x}} \\ &- \frac{(\omega_f^2 H[\dot{x}]-H[\omega_f^2]\dot{x})x}{xH[\dot{x}]-H[x]\dot{x}} = \omega_0^2 - f_1(h_f) - f_2(\omega_f^2), \end{aligned} \tag{6}$$

$$\begin{aligned} 2h_s = &\frac{pH[x]-H[p]x}{H[x]\dot{x}-xH[\dot{x}]} - \frac{\ddot{x}H[x]-H[\ddot{x}]x}{H[x]\dot{x}-xH[\dot{x}]} - \frac{(2h_f H[x]-H[2h_f]x)\dot{x}}{H[x]\dot{x}-xH[\dot{x}]} \\ &- \frac{(\omega_f^2 H[x]-H[\omega_f^2]x)x}{H[x]\dot{x}-xH[\dot{x}]} = 2h_0 - f_3(h_f) - f_4(\omega_f^2). \end{aligned} \tag{7}$$

Eqs. (6) and (7) can be rewritten as $\omega_0^2 = \omega_s^2 + f_1(h_f) + f_2(\omega_f^2)$ and $2h_0 = 2h_s + f_3(h_f) + f_4(\omega_f^2)$. These relationships indicate that the fast-varying damping and stiffness coefficients both affect ω_0^2 or $2h_0$. Therefore, the initial instantaneous parameters in Eq. (5) are distorted from a mechanical/physical point of view. Since the stiffness and damping coefficients associated with the external loads, ω_{0p}^2 and $2h_{0p}$ are known, their effects can be lumped into the slow-varying stiffness and damping coefficients as follows:

$$\begin{cases} \omega_{0x}^2 = \omega_0^2 - \omega_{0p}^2 = \omega_s^2 - \omega_{0p}^2 + f_1(h_f) + f_2(\omega_f^2) = \omega_{s'}^2 + f_1(h_f) + f_2(\omega_f^2) \\ 2h_{0x} = 2h_0 - 2h_{0p} = 2h_s - 2h_{0p} + f_3(h_f) + f_4(\omega_f^2) = 2h_{s'} + f_3(h_f) + f_4(\omega_f^2) \end{cases}. \tag{8}$$

AMD is an effective low-pass filter for time-varying signals. It employs a time-varying bisecting frequency, $\omega_b(t)$, that separates the low- and high-frequency components in reference to $\omega_b(t)$ at each time. Here, let the remaining low-frequency and the removed high-frequency components of a general signal $s(t)$ with a time-varying bisecting frequency of $\omega_b(t)$ be $AMD_{\omega_b(t)}\{s(t)\}$ and $\overline{AMD}_{\omega_b(t)}\{s(t)\}$, respectively [24]. The bisecting frequencies between the slow- and fast-varying stiffness