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Elastic properties of rhombic mesh structures based on computational homogenisation

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ABSTRACT

Flat mesh structures are used in a wide variety of applications. In particular, meshes with a rhombic unit cell are frequently employed due to their simplicity and relative ease of manufacture. This paper studies the in-plane elastic properties of such a structure as a function of the geometrical parameters by means of homogenisation techniques. We compare predicted elastic in-plane properties (i) including only bending mode of the struts, cf. Gibson-Ashby model, (ii) including both bending and stretching modes of the struts, obtained by homogenisation using beam elements and (iii) by homogenisation using beam-spring elements accounting additionally for strut joint deformation, and (iv) numerical results of elastic properties obtained by homogenisation using solid elements. The expressions of the predicted elastic properties are presented in analytical form. The homogenised elastic properties accounting for both bending and stretching matches very well with those from the model including only bending. The axial deformation of struts thus has negligible impact on the overall elastic behaviour. The complex deformation in the strut joint was also captured in the homogenised using beam-spring elements, and the results agree better with the solid element results. It is concluded that a finite-element-based homogenisation approach could serve as a straightforward analytical method to obtain elastic properties of mesh structures. This approach automatically includes all deformation mechanisms as opposed to the classical unit cell analyses of bending beams.

1. Introduction

Two-dimensional mesh structures, such as lattice truss materials composed of periodic unit cells, possess some useful properties like high stiffness-weight and high strength-weight ratios [1,2]. Moreover, the regularity of mesh structures enables efficient manufacturing processes at lower costs compared to more complex and irregular structures. With the development of improved manufacturing technologies they are becoming more widely used in the fields of civil engineering [3,4], aeronautical engineering [1,2], additive manufacturing [5], and even in medical applications [6,7] and photo-electronic devices [8]. For example, a comprehensive study has been carried out to develop a complete application of using expanded metal panels to upgrade reinforced concrete moment resisting frames under seismic actions [4]. Crest-to-crest wave springs [9,10], which consist of periodic cells, have been developed as an alternative to normal compression springs. Tubular implants with a coupled helical coil structure [11] and mesh-structured stents [12] have been manufactured with the eventual aim to replace the abnormal tubular organs in human body. To make use of the mechanical advantages of mesh structures, it is essential to characterize and understand their structural properties. Considering the regularity and relative simplicity of mesh structures, one could expect that it is possible to derive analytical expressions for the structural properties, especially for elastic properties, which have the advantage of expedient parametric investigations in the design process compared with more detailed numerical approaches.

For most mesh structures, the mechanical properties could be derived based on the periodicity of the structure. The methods that are most frequently employed are analytical and computational homogenisation. A considerable number of studies were carried out to predict the equivalent elastic properties of periodic mesh structures, e.g. honeycomb structure e.g. [13–17], grid and lattice structures e.g. [18–20], and beam-like structures e.g. [21,22]. However, to our knowledge, there is still a lack of analytical studies presenting the closed-form solution and its application window on the effective elastic properties of mesh structures with a rhombic cell. This structure is shown in Fig. 1(a), which can be regarded as two sets of an infinite number equidistant parallel beams fused together at the intersections. Though

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Nomenclature

a	ratio *of* strut diameter to length $(=D/L)$
$\mathbf{A}$	matrix in beam-spring element homogenisation
$\mathbf{C}$	effective stiffness matrix of mesh structure
D	cross-sectional diameter of struts
E	Elastic modulus
F	force on strut associated with the applied stresses
G	shear modulus
I	area moment of inertia of struts $(= \pi D^4/64)$
$\mathbf{I}$	second order unit tensor
$\mathbf{K}$	stiffness matrix of RVE
L	strut length in rhombic RVE
$\mathbf{M}$	matrix composing the matrix $\mathbf{C}$ of beam-spring element homogenisation
M	moment on strut associated with the applied stresses
$\mathbf{n}$	unit normal vector of the RVE boundary
$\mathbf{P}$	total nodal force vector of RVE
$\mathbf{u}$	displacement vector of RVE points
$\mathbf{U}$	total nodal displacement vector of RVE
V	volume of the RVE
Γ	boundary of the RVE
$\mathbf{X}$	position vector of RVE points in the reference configuration
$\mathbf{x}$	position vector of RVE points in the deformed configuration
α, l, h, b	geometrical parameters of honeycomb RVE
φ	intersection angle between struts
ν	Poisson ratio
σ, τ	stress
ε, γ	strain
δ	end deflection of strut

Superscripts

$+$, $-$	corresponding nodes on opposing boundaries of RVE
i	node number in the RVE model

Subscripts

M	macroscopic quantities
m	microscopic quantities
1	longitudinal
2	transverse

meshes like isogrid or diamond lattices have superior properties, it is emphasized that the simplicity of rhombic meshes makes them easier to manufacture and cost efficient compared with more complex mesh structures. For instance, since the struts are aligned in adjacent unit cells, continuous filaments can readily be used in manufacturing process, such as in filament winding [11]. Comprehensive studies regarding expanded metal panels, one type of rhombic mesh structure, have been carried out both theoretically and experimentally in [4], and the focus was mainly on the shear behaviours under both monotonic and cyclic loadings. An important study on analytical homogenisation of elastic meshes has been presented by Hohe et al. [19]. Their homogenisation method is likewise based on energy equivalence for beam elements considering the periodicity of certain meshes. In the present work, we specifically address rhombic meshes (frequent in applications) and compare the results with a full 3D solid element solution as an accurate reference. The boundary conditions are periodic, which is considered more realistic than assuming a homogeneous strain field. Furthermore, the effect of the non-negligible joint volume is also taken into account.

Given increased use of mesh structures, it is of interest to develop and compare different ways to efficiently estimate engineering elastic properties which are more convenient than full finite-element solutions. The comparisons should then include the limitations of different methods and which underlying deformation mechanisms are more important than others for accurate predictions. These engineering incentives have prompted the present investigation.

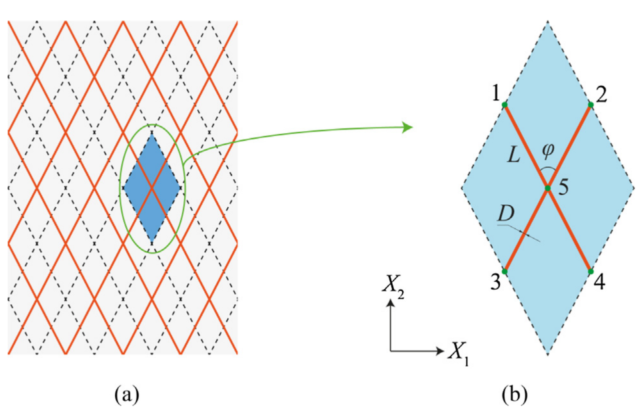

Fig. 1. Schematic illustrations of (a) the periodic mesh structure, and (b) the geometry of the RVE. The solid lines delineate the mesh structure, and the dashed black lines RVE boundaries.

In present work, four methods: (i) a beam bending model, (ii) a more straightforward closed-form solution from beam element homogenisation, (iii) a similar implicit solution from beam-spring element homogenisation, and (iv) a more realistic solution from solid element homogenisation were developed, with the aim of finding an equivalent continuum description of a mesh structure with a rhombic representative volume element (RVE) and showing the application window of homogenisation method through comparison. Despite a two-dimensional analysis, we employ the conventionally used term RVE, since the rhombic mesh can be extruded into a third direction, forming e.g. a sandwich core material with a rhombic cross section. Analytical expressions of the effective elastic properties were obtained by the homogenisation method and the complex deformation of the strut joint in the RVE was approximately captured in the analytical approach.

This work is organized as follows. In Section 2 the geometry of the RVE, shown in Fig. 1(b), and boundary conditions for homogenisation are introduced. Then, four different approaches are developed for the homogenisation of the rhombic RVE. Section 3.1 analyses the rhombic RVE using Euler-Bernoulli beams, which neglects the axial deformation of the struts. In Section 3.2, the RVE composed of beam elements is homogenised using computational homogenisation, where beams also account for axial deformation. Analytical formulas for the effective elastic properties of the equivalent continuum are obtained. In Section 3.3, a more detailed RVE composed of beam and spring elements, accounting also for strut joint deformation, is homogenised analogously. Implicit expressions for the associated effective elastic properties are determined as well. Subsequently in Section 3.4, computational homogenisation is implemented again with a RVE using realistic solid elements. Effects of triaxial stresses are then included, which are expected in the joints. The influences of geometrical parameters of the mesh structure on the effective elastic properties are parametrically investigated in Section 4 for the four homogenisation methods. It is shown that the proposed homogenisation procedure is versatile in the sense that it can be applied to arbitrary RVEs, and that analytical expressions can be straightforwardly obtained where axial deformation of the beams and triaxial deformation of the strut joint are included. Practical issues of stiffness design are also addressed, e.g. by controlling strut cross-section, distance between joints, intersection angles etc. in