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Finite-element implementation for nonlinear static and dynamic frame analysis of tapered members

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ABSTRACT

Non-prismatic members are popular for civil engineering structures. This paper derives a set of unified beam-column formulations for nonlinear static and dynamic analyses of the structures made of members with tapered sections, addressing the problems in engineering design practices. The element shape-functions are established upon the local-axes by extracting the rigid-body movements for simplifying mathematical expressions. To represent the variations in the stiffness gradients of tapered sections, the tapered-variability indexes are proposed. The generalized tangent stiffness and consistent mass matrices are developed based on the indexes. When analyzing non-prismatic members, the conventional method for handling member loads is inapplicable because it is derived for prismatic sections. Therefore, a new approach for converting the member loads acting on tapered members into the equivalent nodal forces is proposed based on the energy conservation principle. To consider the offsets in section axes, the eccentricity matrices are employed. For allowing large deflections, the incremental secant-stiffness method (ISM) based on Updated-Lagrangian (UL) description is proposed. Finally, extensive examples are provided for validating the accuracy and efficiency of the proposed element formulations in solving both the static and dynamic nonlinear problems.

1. Introduction

Steel structures using tapered sections are becoming popular with the applications of the Building-Information-Modeling (BIM) and the Automated Robotic Welding (ARW) technologies that make the fabrication of non-prismatic members from steel plates economical and efficient. Further, high-strength steel is mostly available in the form of plates so that fabrication is sometimes unavoidable, and there is no reason to weld prismatic members to take commonly non-uniform bending moments. Recently, one of the world's longest span single-layer domes (see Fig. 1) was constructed in Macau and designed by the authors using tapered members with the direct analysis method (DAM).

Experimental studies on the behaviors of tapered members were conducted by several researchers. For example, Butler and Anderson [1,2] studied the elastic flexure and lateral buckling resistance of web-tapered beams. Prawel et al. [3] investigated the inelastic bending and buckling strengths of fifteen web-tapered beams. Salter et al. [4] tested the inelastic stability of tapered columns. Shim et al. [5] examined the flexural performances of four rafters with tapered I-section. Recently, Wang et al. [6], Hong and Uang [7], Yang et al. [8] and Su et al. [9] conducted the experiments for the portal frames with non-uniform members. Their experiments revealed that failures of tapered members were mostly caused by buckling.

Based on the experimental observations, other researchers developed empirical design methods for determining the strengths of tapered members. For example, the equations for checking the flexural buckling of web-tapered I section columns were proposed by Galambos [10], who revised the traditional effective length method by introducing a modification factor. Shiomi and Kurata [11] developed an interactive formula consisting of the axial forces and moments for determining the ultimate strength of tapered-I and box sections using the statistical technique. Polyzois and Raftoyiannis [12] evaluated the equivalent length factors for lateral torsional stability of non-prismatic I beams. Andrade and Camotim [13] studied the lateral torsional buckling behavior of mono-symmetric I beams. Trahair [14–16] analyzed the bending and torsional behavior of tapered beams. Some design codes and guidelines, such as AISC Guideline [17] and GB51022-2015 [18], give the empirical equations for stability checks of the web-tapered members in portal frames. Eurocode 3 [19] provides the general equations for non-prismatic members; however, some coefficients in the equations are needed to be pre-determined through a sophisticated finite-element analysis (FEA).

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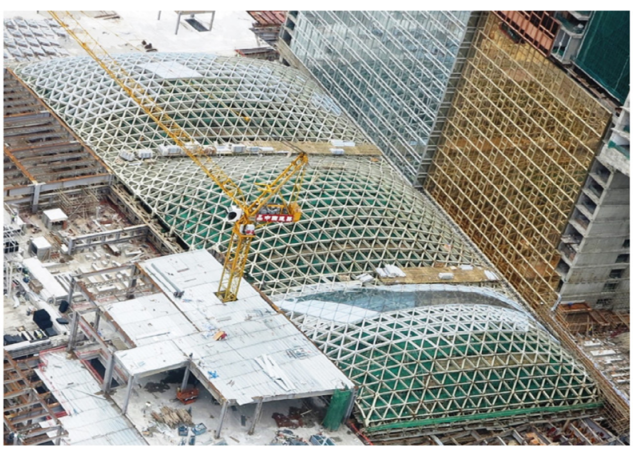
The following symbols are used in this paper:

$\overline{A}$	equivalent section area
$A(x)$	the expression of the section area along the element length
$\boldsymbol{B}^{(m)}$	strain-displacement matrix of the element m
$\boldsymbol{C}(\xi)$	the conversion matrix based on Hooke's law
E	Young's modulus
$\widehat{\boldsymbol{F}}$	equivalent nodal force vector
$\widehat{\boldsymbol{F}}_O$	force vector acting at the geometric centroid of the section
$\boldsymbol{R}^i$	nodal virtual force along the i^{th} degree of freedom
G	shear modulus
$I(x)$	the expression of the moment of inertia along the element length
I_L	the moment of inertia at the element's left end
I_R	the moment of inertia at the element's right end
I_y	moment of inertia about y-axis
I_z	moment of inertia about z-axis
$\overline{J}$	the averaged torsional constant of a non-prismatic element
J_i	torsional constant at the i^{th} interval points
$\boldsymbol{K}_e$	element stiffness matrix
$\boldsymbol{K}_E$	element stiffness matrix after considering the eccentricities of section axes
$\mathrm{K}_{j,i}$	tapering stiffness factor in the element stiffness matrix
$\boldsymbol{L}$	transformation matrix for establishing the relations from the element local to element intermediate axes

L	element length
$\boldsymbol{M}_e$	consistent mass matrix for non-prismatic element
$\boldsymbol{M}_P$	consistent mass matrix for prismatic element
$\boldsymbol{N}$	stiffness matrix for rigid body movements
$\boldsymbol{N}(\xi)$	displacement interpolation matrix
n_A	variation index for cross-section area
n_I	variation index for flexural rigidity
$\boldsymbol{T}$	transformation matrix for establishing the relations from the element intermediate axis to the global coordinates
$\boldsymbol{T}_D$	eccentricity matrix for nodal displacements
$\boldsymbol{T}_F$	eccentricity matrix for member forces
$\widehat{\mathbf{U}}$	shape function vectors in the element local axes
$\overline{\boldsymbol{U}}^i$	nodal virtual displacement of the i^{th} degree of freedom
$\hat{\boldsymbol{u}}$	vector of the independent nodal displacements in the element local axis
$\widetilde{\boldsymbol{V}}$	non-dimensional transfer vector of the equivalent nodal forces
$\boldsymbol{\varepsilon}^{(m)}$	strain of the element m
ρ	density
γ	correction matrix for the consistent mass matrix
$\boldsymbol{\tau}^{(m)}$	stress of the element m
υ_A	$\sqrt[n_A]{A_r/A_l}$
υ_I	$\sqrt[n_I]{I_r/I_l}$
ξ	non-dimensional coordinate position: $(x + \mathrm{L}/2)/L$



(a) Overall View

(b) Authors standing in front of the dome

Fig. 1. A space dome constructed by tapered I-section members.

Currently, two simplified approaches are widely adopted in practices to model non-prismatic members in frame analyses, i.e., the stepped-element and the approximated-stiffness methods. The former method is based on the finite-element theory, utilizing several segmented prismatic elements to simulate a tapered member. Empirically, in order to obtain an accurate result, at least 20 sub-elements are required to simulate one non-prismatic member [20], which significantly increases the computing effort. An alternative approach is to simplify the variations of flexural rigidities to be linear, parabolic or cubic along the member length, which has been used by the researchers such as Banerjee and Williams [21,22], Valipour and Bradford [23], Mohaamad et al. [24,25] and Gere and Carter [26]. The accuracy of the approximated-stiffness method is sometimes questionable because it is derived based on the empirical assumptions. Recently, Liu et al. [20,27] proposed the tapered elements using the analytical expressions of the stiffness variations. Their formulations are very accurate for analysis but complicated for computer programming, which requires different stiffness matrices for various section types. Thus, this paper derives the unified beam-column formulations for covering common types of tapered sections.

In a standard procedure of FEA, the loads applied along the member length, e.g. uniform distributed load (UDL), need to be converted into the nodal forces for numerical analysis [28–30]. When analyzing tapered members, this conventional method for handling member loads on prismatic members is inapplicable because the equivalent forces are derived for uniform members and would be affected by the stiffness variations. Therefore, Attarnejad et al. [31] proposed an algorithm for calculating the nodal forces of member loads for tapered elements, where the shape functions based on a flexibility matrix are used. However, their method needs an iterative procedure at the element-level, causing additional computing expenses. Murín and Kutiš [32] proposed a series of "transfer functions" to calculate the nodal equivalent forces. Some researchers, such as Luo et al. [33] and de Araujo and Pereira [34] utilized the virtual work principle to determine the equivalent nodal forces, where the expressions of the nodal forces are conjunct with the element stiffness matrix, making the formulations very complicated to be used. Therefore, the method for handling the member loads acting on a non-prismatic member is developed in this paper.