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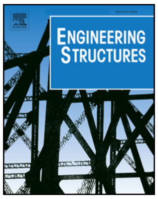

On estimation of seismic damage from ductility and hysteretic energy demands in equivalent oscillators using linear response

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ABSTRACT

Estimation of damage in a structure under anticipated seismic events is important for its performance-based design. This can be done in terms of the ductility and hysteretic energy demands for each of the anticipated plastic hinges under the anticipated ground motions. This study considers the possibility of quantifying the structural damage simply from the linear analysis based on the elastic design spectra of the ground motions and undamped mode shapes of the structure. A two-parameter model is first developed for the estimation of hysteretic energy demand in single-degree-of-freedom (SDOF) oscillators for five types of nonlinearities from the linear displacement peaks. The parameters of this model are estimated for various initial periods, nonlinearity types, and specific values of damping ratio, maximum possible ductility demand, and hysteretic parameters. Next, it is assumed that damage in each of the equivalent oscillators corresponding to different modes of vibration of the structure can be combined to quantify the structural damage. The hysteretic properties of these equivalent oscillators are estimated in the cases of 2-DOF and 3-DOF frames, and linear-peaks-based models for ductility demand and hysteretic energy demand are then used to estimate damage index for each of these oscillators. Finally, a combination rule is proposed to suitably combine these damage indices and thus estimate the extent of overall damage. A numerical study with the help of a suite of 100 ground motions illustrates how the proposed methodology estimates the damage levels of 2-DOF and 3-DOF example frames with strain-hardening bilinear and stiffness-degrading Riddell-Newmark type nonlinearities in the moment–curvature relationships of their column sections.

1. Introduction

A structure may be designed to behave linearly, leading to little or no damage, under the ground motions consistent with the perceived seismic hazard level at the site of the structure. Such design practice, however, does not make use of the ductility typically available in duly designed and detailed structures. This leads to high construction costs of the structures and is thus suitable only for those structures where no structural damage is to be permitted due to the disastrous consequences of any damage. Hence, it is more common to allow the structures to undergo plastic deformations and suffer the maximum possible damage, causing no danger to the lives of their inhabitants, during the most severe ground motions. For this, one needs to ensure that, in the case of performance-based seismic design, the extent of structural damage is properly estimated during the most severe ground motions and is then checked for not exceeding the specified limits.

Seismic damage is usually quantified through a damage index which represents degree of damage on the scale of 0–1, with 0 implying no damage and 1 representing the collapse of the structure. Damage index for a single-degree-of-freedom (SDOF) system may be expressed as a linear weighted combination of the contributions of maximum displacement (through ductility demand ratio) and energy dissipation during the repeated cyclic loading (through hysteretic energy demand) [1,2]. For multi-degree-of-freedom (MDOF) systems, an overall damage index may be calculated from the individual damage indices corresponding to different localized sources of plastic deformations to estimate the degree of damage for the entire system [3]. It is computationally intensive and often inconvenient for a designer to carry out the nonlinear analyses of a given structural system (for the most severe ground motions) required for the estimation of damage index. It is far more convenient to use the results of linear analysis, commonly available in the form of response spectra of the anticipated ground motions, in order to arrive at the reasonably accurate estimates of damage index.

In the case of SDOF systems, damage index may be evaluated without carrying out a nonlinear analysis, provided linear response to the given ground motion can be used for estimating the maximum

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nonlinear displacement and hysteretic energy dissipation during the motion. Several attempts have been made in the past to estimate the maximum nonlinear displacement of a SDOF oscillator from its maximum linear displacement. Some studies considered the use of equivalent linear oscillator together with the elastic displacement spectrum and proposed formulations for equivalent stiffness and damping [4–6]. Most studies have however considered the estimation of maximum nonlinear displacement via inelastic displacement ratio C, which is defined as the ratio of the maximum nonlinear to maximum linear displacement of the oscillator mass. A few studies showed for elastic-perfectly-plastic (EPP) oscillators that at long periods C becomes equal to one [7,8]. This is popularly known as the ‘equal-displacement rule’. Later, the period values above which the ‘equal-displacement rule’ could be applicable have been proposed [9–12]. Some studies [13–15] proposed (a) $C_\mu(T)$ spectra to show the variations of C with the initial period of the oscillator for different values of ductility demand ratio μ, and (b) $C_R(T)$ spectra to show the variations of C with the initial period of the oscillator for different values of response reduction factor R. Shimazaki and Sozen [16] observed that at small periods, a significant variation in C with R occurs. Whittaker et al. [17] showed that $C > 1$ for $R > 5$ or for $T < 1$ s. Some studies [18–21] determined $C_R(T)$ spectra using R–μ–T relations. Miranda [22] observed that this method underestimates the maximum nonlinear displacement. Banerjee [23] proposed $C_R(T)$ spectra based on the normalized relative velocity spectrum of the ground motion under consideration.

There have also been a few studies [24,25] that focused on estimating the ductility demand ratio and hysteretic energy demand from the maximum as well as higher-order peaks of the linear displacement response. Sadhu [24] considered all peaks exceeding the yield level and proposed a single-parameter model to estimate μ from a functional of these peaks in the case of EPP and stiffness-degrading Riddell-Newmark [10] oscillators. The parameter of the proposed model is governed by the initial period, damping ratio and type of nonlinearity in the oscillator. Alluri [25] however demonstrated that the functional proposed by Sadhu [24] would give better correlation if it considered only the largest peak of the linear response. Further, Alluri [25] obtained the values of the parameter of the modified model for five different types of nonlinearities. In comparison with the $C_R(T)$ spectra-based methods of estimating maximum nonlinear displacement, the model proposed by Alluri [25] does not involve ground motion characteristics and is thus simpler to use. Alluri [25] also proposed a model to estimate hysteretic energy demand for five types of nonlinearities in terms of all the linear excursions of the yield level and a parameter governed by the initial period, damping ratio and type of nonlinearity in the oscillator. However, this model gives the same value of normalized hysteretic energy, irrespective of the period and type of nonlinearity of the oscillator, for the unit value of the proposed functional (of the linear excursions of the yield level) and thus needs to involve more than one parameter for conformity with such boundary conditions.

In the case of MDOF systems, Chopra and Goel [26] proposed an approximate procedure to estimate the nonlinear displacement response by neglecting the coupling of modal response coordinates, and then by solving the uncoupled (nonlinear) equations for each ‘modal’ coordinate. Browning et al. [27] and Yaghmaei-Sabegh et al. [28] proposed to estimate the nonlinear displacement demand of such systems by analyzing linear SDOF systems with an effective period and damping ratio. However, these studies have so far been limited to estimating only the nonlinear displacement demand, and no developments have been reported so far in the direction of estimating damage index without requiring any nonlinear analysis.

In the present study, the functional and model proposed by Alluri [25] for hysteretic energy demand are modified for better boundary-condition compatibility and for a more reasonable estimation by considering only the largest few linear excursions of the yield level. A suite of 225 recorded accelerograms, six different yield displacement levels of oscillator, and five different types of oscillator nonlinearities are considered for each initial period of oscillator and assumed damping ratio of 5%. Further, this study attempts at estimating the degree of structural damage in the shear-frame and non-shear-frame types of MDOF systems by estimating damage indices in the equivalent oscillators corresponding to the undamped modes of vibration and then by suitably combining those through a combination rule. A numerical study based on 100 recorded accelerograms and 2-DOF and 3-DOF systems with strain-hardening bilinear and stiffness-degrading Riddell-Newmark type [10] local moment-curvature nonlinearities is carried out to illustrate the proposed methodology.

In the following sections, the single-parameter model proposed by Alluri [25] to estimate hysteretic energy demand is first modified to include one more parameter and to consider the role of only a few largest linear excursions. Following this, the hysteretic properties of the equivalent oscillators corresponding to the vibration modes of example 2-DOF and 3-DOF frames are evaluated and damage index is estimated for each of these oscillators from the linear response in the corresponding mode to the applied ground motion. Finally, a combination rule is developed for estimating damage index for the entire frame structure from the damage indices of the equivalent oscillators.

2. Correlation between hysteretic energy and linear peak response

The hysteretic energy demand of an oscillator subjected to base excitations is one of the two damage parameters considered by Park and Ang [1] and this characterizes the effect of repeated cyclic loading in structural damage. Alluri [25] attempted to estimate this parameter from the linear response of the oscillator by (a) considering a functional in terms of the linear excursions beyond the displacement yield level in the case of single-degree-of-freedom (SDOF) oscillators, and by (b) relating the functional to the hysteretic energy demand through a parameter that depends on the initial natural period and the type of nonlinearity of the oscillator. In this section the model proposed by Alluri [25] is modified for a better adherence to the boundary conditions in the relationship between the functional and hysteretic energy demand. The proposed model involves a new parameter which also depends on the initial natural period and the type of nonlinearity of the oscillator. This parameter is estimated and modeled as a function of the initial natural period for five oscillators with different types of nonlinearities.

2.1. Proposed model

The functional considered by Alluri [25] for estimating the hysteretic energy demand of a SDOF oscillator is expressed as

$$f = \sum_{i=1}^{n_e} \frac{X_i - X_y}{X_y} \tag{1}$$

where X_y is the specified yield displacement level, and $X_i(\geqslant X_y)$ is the ith largest peak amplitude in the absolute linear displacement response $|X(t)|$ of the oscillator. Further, $n_e(\leqslant N_e)$ is the selected number of the largest (linear) excursions of the yield displacement level by the oscillator response to the ground motion, where N_e represents the total number of linear excursions beyond the yield level. The value of n_e is so chosen that the functional f has the maximum correlation with the hysteretic energy demand E_h of the oscillator (as obtained from the inelastic excursions of the yield displacement level during the ground motion). The functional f is assumed to be related to the hysteretic energy demand E_h as [25]

$$E_h^* = f^p \tag{2}$$

where E_h^* represents the normalized hysteretic energy defined as

$$E_h^* = \frac{E_h}{F_y X_y} \tag{3}$$

Here, F_y denotes the yield force level and p is a parameter that depends