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Identification of transient vibration characteristics of pile-group models during liquefaction using wavelet transform

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ABSTRACT

A time–frequency approach based on the wavelet transform is used to examine the transient vibration characteristics of two 2 × 2 pile-group models tested in a shake table. The models are subjected to three different records consisting of white noise input and two differently scaled records from the 2011 Christchurch Earthquake. In contrast to conventional Fourier analysis, the proposed method has the advantage of enabling the visualisation of the temporal variation in structural frequencies and frequency content of ground motion due to liquefaction in an effective way. It is found that liquefaction causes a decrease in structural frequency, whose reduction depends on the rate of excess pore pressure build-up, whereby high rates ("fast liquefaction") lead to greater reduction, ie, up to 51%. Liquefaction is also responsible for the elongation of the predominant period of the ground motion and narrowing of its overall frequency bandwidth. The combined effect of reduction in structural frequency and filtering of high frequency components of the ground motion may lead to moving resonance condition, resulting in amplification of structural response. After the onset of liquefaction, there is a redistribution of maximum bending moment toward deeper elevations, indicating that kinematic soil-structure interaction dominates the overall seismic response.

1. Introduction

Extensive damage to pile-supported structures, particularly bridges, high-rise buildings, coastal structures, have been observed in liquefiable soils after most major earthquakes, including recent events in India [11], China [24], Japan [5,16], Italy [31], Nepal [9] and New Zealand [37]. Because damage to foundations occurs beneath the ground, it is difficult to ascertain what the actual mechanism of failure is unless deep excavations and/or integrity tests are carried out to determine damage patterns. Observations made during post-earthquake reconnaissance missions at sites affected by liquefaction have highlighted that cracks and evidence of plastic hinge formation can be found at various depths along piles [2,14,18,47]. Such findings are somewhat surprising considering the conservative approach adopted in routine practice according to which piles are designed to remain elastic even during strong ground shaking, and ultimate limit state verifications [13]. Evidently, studying the seismic response of piled foundations is challenging due to the non-linear behaviour of the soil, and the complex nature of the dynamic soil–structure interplay. Moreover, in piles passing through saturated cohesionless soil, the analysis is further complicated since the shaking leads to development of excess pore pressure, resulting in a temporary reduction of stiffness and strength of the foundation soil. This phenomenon, commonly referred to as soil liquefaction, leads to further non-stationarity and non-linearity, including filtering effect of the liquefied deposit to seismic waves, which results in a temporal shift of the frequency content of the shaking towards lower frequencies, and temporal variation of the vibration characteristics of the foundation-structure system. Indeed, the combined variation of ground motion's frequency content and structural vibration characteristics due to liquefaction may have important implications on the seismic demand and capacity of pile-supported structures. Lombardi and Bhattacharya [32] concluded that the lengthening of fundamental period due to excess pore pressure build-up leads to a progressive reduction of base shear force, owing to lower spectral accelerations at high periods. To quantify the variation in vibration characteristics of pile-supported structures caused by soil liquefaction, Lombardi and Bhattacharya [31] applied a Fourier spectral analysis to acceleration responses of models subjected to a white noise input applied by means of a shake table. To reduce the velocity of the liquefaction front, the amplitude of the input motion was gradually

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incremented over 300 s. Notwithstanding the restrictions of the Fourier spectral analysis when applied to non-stationary time series (for further details, see [46], the Fourier transform was applied to windows within which the data was deemed to be stationary. Evidently, the validity of piecewise stationarity depends on the size (ie, duration) of the window, which has to be greater than the lowest period of the mode of interest, and long enough to obtain an acceptable frequency resolution. Evidently, this method is applicable to relatively long duration time series, yet it might be questionable whether such an approach still valid to dataset obtained from time histories recorded in real earthquakes.

The wavelet transform is a convenient mathematical tool for the analysis of highly non-linear and time-dependent processes, such as those encountered in soil liquefaction. In this paper, we will use the wavelet transform to investigate the temporal variation in frequency characteristics of piled foundation models caused by increase in excess pore pressure, which eventually leads to liquefaction condition when the excess pore pressure equalises the overburden stress. After a brief overview of the different methods available for the analysis of strong ground motion data, the basics and main applications of the wavelet transform to structural and geotechnical problems are presented. The method is subsequently applied to data obtained from a series of shake table tests with the aim to quantify the temporal variation in vibration characteristics of two 2 × 2 pile-group models. The wavelet transform is first applied to free vibration and white noise tests previously analysed by Lombardi and Bhattacharya [31] by means of a conventional Fourier analysis. This not only provides a way to verify and validate the proposed methodology, but it also gives authors the opportunity to compare the findings obtained from the Fourier and Wavelet transforms. Subsequently, the method is applied to two tests wherein models were subjected to two differently scaled records from the 2011 Christchurch earthquake. The final presentation is a time-frequency distribution, designated wavelet spectrum. The second part of the paper investigates the implication of the change in frequency characteristics on the seismic demand and capacity of the models, expressed in terms of base shear, and time histories of bending moment and displacements, respectively. The paper concludes with a discussion on the practical implications of the research findings on the seismic design of pile-supported structures in liquefiable soils.

1.1. Spectral analysis of strong ground motion data

The analysis of strong ground motion data, whether from physical measurements or numerical modelling, most likely exhibits the following issues: (i) limited time span of data with meaningful information, ie, short records with high signal-to-noise ratios; (ii) intrinsic non-stationary of the data; and (iii) intrinsic non-linearity of the process being recorded. The Fourier spectral analysis has been widely used for computing the energy-frequency distribution of strong ground motion data, although the method is strictly applicable to linear systems and time-series characterised by ergodicity and stationarity. In earthquake engineering, the conditions of linearity and ergocity are rarely met since most of the available data is transient in nature and intrinsically non-linear. The requirement of stationarity may be satisfied by assuming that data is stationary within a limited time span (ie, piecewise stationarity), or data becomes stationary when time approaches infinity (ie, asymptotically stationarity). When the Fourier spectral analysis is applied to data that do not satisfy these assumptions, the Fourier transform introduces spurious harmonic components that artificially widen the frequency spectrum. A number of data processing methods for non-linear and non-stationary processes are available, including: *(i)* Wigner-Ville distribution method, which is widely used by the electrical engineering community [10], *(ii)* empirical orthogonal function expansion, a method popular in remote sensing, with applications in oceanography and meteorology research [42], *(iii)* Hilbert Huang method [20–22], which has been applied to a number of geophysical data and used for atmospheric and climate studies [23]; and *(iv)* Wavelet transform [35], whose applications to earthquake, wind and ocean engineering research can be found in Gurley and Kareem [17]. Of particular relevance to the current study is the application of wavelet transform to the analysis of the energy-frequency distribution of earthquake records [25,39,45] and spectral non-stationary due to propagation of seismic waves through soft deposits [1,8,36].

Evidently, each of these methods has its own advantages and disadvantages, whose implications on the analysis of non-linear and non-stationary processes induced by soil liquefaction deserve a separate discussion. This, however, would be out of the scope of this paper, hence it is omitted herein. Yet, for the interested reader who wishes to revisit the present study by evaluating the performance of different data processing methods, the research data supporting this publication is publically available as supplementary information at http://dx.doi.org/10.15127/1.296929. This study adopts the wavelet analysis because of its proven application for the analysis of soil liquefaction [44] and evaluation of change in vibration characteristics of structures founded in soft deposits during strong shaking (Naga and Eatherton, 2013). The following section briefly presents the mathematical formulation of the wavelet transform.

Table 1
Geometrical and mechanical properties of the pile-group models.

Model ID	Outer diameter [mm]	Thickness [mm]	*EI* pile [Nm^2]	Pile-cap mass [kg]	Natural frequency [Hz]
GP1	25.40	0.71	294	13.08	1.18
GP2	41.28	0.71	1305	22.72	1.86

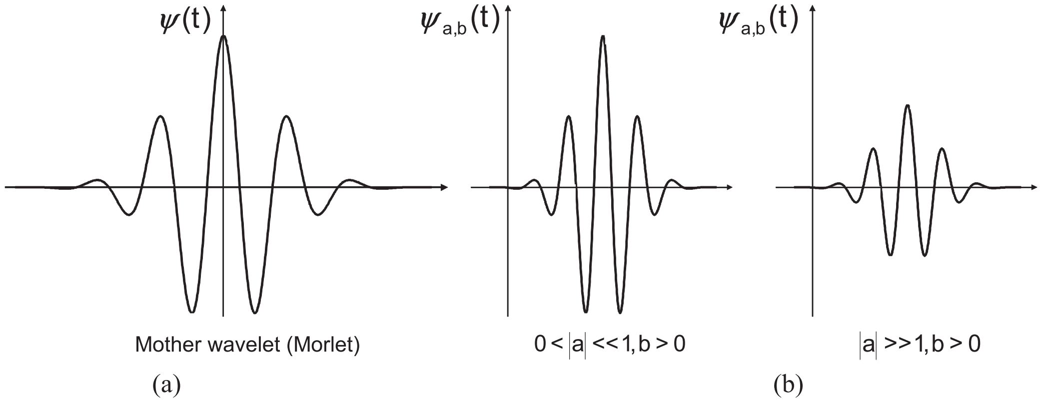

Fig. 1. Qualitative representation of an example of mother wavelet: (a) Morlet wavelet; (b) compressed and dilated versions of the mother wavelet.