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An axiomatic/asymptotic evaluation of the best theories for free vibration of laminated and sandwich shells using non-polynomial functions

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ABSTRACT

This paper presents Best Theory Diagrams (BTDs) constructed from non-polynomial terms to identify best shell theories for the free vibration analysis of laminated and sandwich shell. The shell theories have been constructed using Axiomatic/Asymptotic Method (AAM). The refined models are implemented following the compactness of a unified formulation developed. The governing equations are derived from the Hamilton's Principle. Navier-Type solution technique is used for solving the eigenvalue problem of simply supported shell. The BTDs use 3D equilibrium solutions as a reference. The BTDs built from non-polynomial functions are compared with Maclaurin expansions. The results are compared with Layerwise solutions. Cylindrical and spherical shells with different layer-configurations are investigated. The results demonstrate that the shell models obtained from the BTD using non-polynomial terms can improve the accuracy obtained from Maclaurin expansion for a given order of expansion of a displacement field.

1. Introduction

It is well-known that shells have excellent load-carrying capability in comparison to plates, due to its curvature. In addition, it is well-known that classical composite materials have many attractive properties, due to their strength, high-stiffness-to-weight ratio and remarkable fatigue strength, resistance to corrosion and the design flexibility for desired applications. As a result, composite shell structures meet the demands of many engineering fields such as aerospace, automobile, mechanical, civil and marine.

The development of classical shell theories (CST) were based on the papers by Love [1] and Aron [2] who extended the Kirchhoff hypothesis, valid for flat panels, to shell structures. The main classical contributions are documented in many textbooks such as Timoshenko and Woinowsky-Krieger [3], Flügge [4], Leissa [5], Kraus [6], Gould [7] and Soedel [8]. The CST neglects transverse shear and normal stress and this generates inaccuracies for anisotropic and thin shells. Hildebrand et al. [9] proposed the first order shear deformation theory (FSDT) to include the effect of transverse shear stress. Its displacement field are modeled linearly the displacement through the thickness. More accurate theories are the Higher Order Shear Deformation theories (HSDTs). HSDTs can be expanded using several polynomial [10–12] and non-polynomial [13–15] functions along the thickness direction. A generalization of HSDTs are so-called Unified Formulations (UFs) [16–18]. UFs permits to build any order structural model; moreover, UFs and HSDTs can be implemented in two different approaches: Equivalent Single Layer (ESL) and Layer-wise (LW) [19–21]. An ESL model considers a multilayered structure as a single lamina. In the other hand, the LW approach consider each lamina separately. LW has quasi-three-dimensional capabilities; but presents a higher computational complexity and cost in comparison to ESL. An excellent review on UFs and HSDTs was provided by Caliri et al. [22].

The Carrera's Unified Formulation (CUF) [18] is a fundamental mathematical formulation for compactness and study shear deformation theories from general perspective. According to CUF, the displacement of the shell is defined via an expansion of the thickness coordinate [23]. The governing equations are given in term of a few fundamental nuclei whose expressions do not change by varying the order of expansion. The displacement can be considered to solve different mechanical problems [24,25], thermo-mechanical [26,27], piezo-electric [23] and recently fluid-structure interaction problems [28]. The partial differential equations are solved via Navier type solutions for solving several free vibration problems of cylindrical and spherical multilayered panels.

Asymptotic method [29] defines a suitable kinematics model for a given structural problem by investigating the role played by the shear

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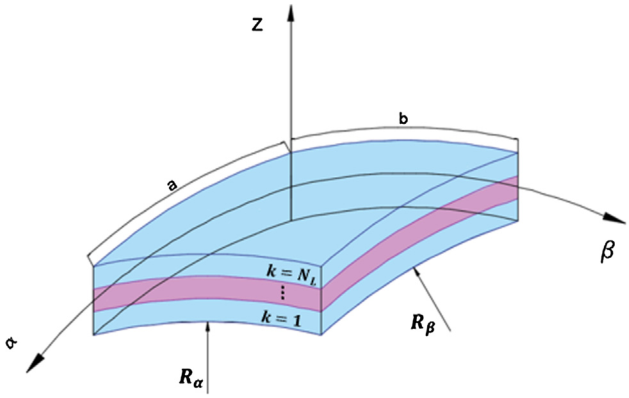

Fig. 1. Spherical shell panel and reference system.

Table 1
Thickness functions.

Model	F_0	F_1	F_2	F_3	F_4
Pol.	1	z	z^2	z^3	z^4
Hyb. 1	1	z	$\cos(\frac{\pi z}{h})$	$\sin\left(\frac{\pi z}{h}\right)$	$\sinh\left(\frac{2\pi z}{h}\right)$
Hyb. 2	1	z	$e^{\frac{2z}{h}}$	$\sin\left(\frac{2\pi z}{h}\right)$	$\sinh\left(\frac{2\pi z}{h}\right)$
Hyb. 3	1	z	$e^{\frac{z}{h}}$	$\sinh\left(\frac{\pi z}{h}\right)$	$\sinh\left(\frac{2\pi z}{h}\right)$

Table 3
Symbol to indicate the status of displacement variable.

Active term	Inactive term
✓	–

strain shape functions in term of a perturbation parameter (type of panel, curvature-radius-to-size and side-to-thickness-ratio). Axiomatic method [30] is related to the results of certain hypothesis based on a conjecture (the cross-section remains plane, shear strains are neglected, etc.). In this framework, the Axiomatic/Asymptotic method (AAM) is a powerful tool to reduce the computational cost. Best theory diagram (BTD) [31] is a curve which determine the minimum number of expansion terms required to meet a given accuracy. Candiotti et al. [32] presented BTD for non-polynomial terms to identify the best plate theories for metallic and functionally graded plates. Carrera et al. [33] studied the BTD for metallic and laminated shells. The present paper introduces for the first time several hybrid reduced models, i.e. BTDs, for the dynamic analysis of multilayered shell problems using non-polynomial expansions.

Tornabene et al. [34] presented a comparison between different methods for solving dynamical problems of shells with constant curvature. The methods used were 2D and 3D finite elements and the generalized differential quadrature method. Brischetto [35] studied the importance of the approximation of the curvature in equilibrium equations for solving free vibration problems of simply supported shells. Brischetto and Torre [36] compared classical 2D finite element and exact three-dimensional solution for finite element and demonstrated that the difference depended on the considered modes, the order of frequency, the thickness ratios of the structure, the geometry, the embedded material, and lamination sequence. Malekzadeh and Baghestani [37] presented a semi-analytical method for free vibration analysis of moderately thick doubly curved open shells with arbitrary geometry and classic boundary conditions. Bouazza et al. [38] proposed an interesting n$^{\text{th}}$-higher-order shear deformation theory with only two unknowns for the study of free vibration of cross-ply and antisymmetric angle-ply laminated plates. Hwu et al. [39] studied free vibration for sandwich cylindrical panel and plate with orthotropic elastic core and laminated faces using a modified FSDT and solved the governing equation using Ritz, Navier and Levy solution. Viola et al. [40] proposed a general HSDT for dynamical analysis of shells and solved the equations numerically by using the generalized differential quadrature technique and differential geometry to define the arbitrary shape of the middle surface of shells with different curvatures. Mantari and Ore [41] developed a novel displacement field which include undetermined integral terms and contains only four unknowns for the study of sandwich laminated plates. Ye [42] proposed a unified Chebyshev-Ritz formulation to investigate vibration of composite laminated open cylindrical, conical and spherical shell. Fazzolari and Banerjee [43] extended the hierarchical trigonometric Ritz formulation using Reissner's mixed variational theorem for solving the free vibration of doubly-curved anisotropic laminated composite shells. AAM shells theories were developed by virtue of a deep study on the effectiveness of each term in both the displacements and transverse stress fields. In [44] free vibration response of laminated composite and sandwich shell is analyzed by means of 2D finite element model based on higher order zigzag theory and incorporating all three radii of curvatures including the effect of cross curvature in the formulation using Sanders' approximations. Their finite element model is claimed to satisfy the inter-laminar shear stress continuity at the interfaces in addition to higher order theory features. Huang and Zheng [45] investigated nonlinear vibration and dynamic response of a simply supported, shear deformable laminated plate subjected to a transverse dynamic loading and having initial in-plane static loads and resting on Pasternak elastic foundation. The Reddy's higher order shear deformation plate theory and general von Kármán-type equation are used for mathematical formulation of the problem,

Table 2
Expansion model representation.

Full model representation

✓	✓	✓	✓	✓
✓	✓	✓	✓	✓
✓	✓	✓	✓	✓

Reduced model representation

✓	-	✓	✓	✓
✓	✓	✓	✓	✓
✓	✓	✓	✓	✓