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Stress field solution for strip loaded reinforced concrete blocks

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ABSTRACT

This paper aims at contributing to a better understanding of the physical-mechanical behaviour of partially loaded areas in reinforced concrete. Following a review of the current state of the art on this topic, which includes a description of the mechanical behaviour and existing predictive models, a remarkable knowledge gap in this standard problem is revealed. A new stress field solution for reinforced concrete blocks under strip loading is developed herein which is based on the lower-bound theorem of limit analysis and adopts a modified Coulomb failure criterion for concrete. Contrary to existing empirical design rules, the new mechanical model consistently accounts for the behaviour of concrete under multiaxial compression as well as the favourable effect of transverse reinforcement. While a comparison of the ultimate loads predicted using the new stress field solution shows a good correlation with the available test data, further experiments are required particularly for the validation of the predicted higher resistances of heavily confined areas as compared to existing design rules.

1. Introduction

Partially loaded areas are a common problem in engineering practice, where concentrated loads are often applied to concrete members over a relatively small area. Although this apparently simple problem has been studied experimentally, analytically and numerically from the early days of reinforced concrete, with seminal tests dating back as far as 1876 (Bauschinger [1]), there is a remarkable knowledge gap regarding the mechanical behaviour of partially loaded areas in reinforced concrete. Most existing approaches, including design rules in international codes, are purely empirical and are based on tests on unreinforced concrete. Some researchers tackled the problem analytically using Elasticity Theory, which allows determining the stress state for low load levels prior to concrete cracking and estimating the position and direction of the first cracks, but fails to predict the structural response after cracking and the associated stress redistribution within the member. A comparison of current design rules with test results shows that they typically yield overly conservative estimates of the ultimate load, particularly for reinforced blocks.

This paper aims at contributing to a better understanding of the physical-mechanical behaviour of partially loaded areas in reinforced concrete. To this end, the fundamental aspects of their structural response are investigated for the simple case of purely axial, concentric loading. A mechanically consistent model for this basic problem is subsequently proposed herein. Additional considerations which include transverse shear and eccentric loading will be treated in future works. Furthermore, the results focus on strip loading since the work presented herein forms part of a broader research programme [2] investigating the structural behaviour of one-way concrete hinges, whose behaviour under axial load essentially corresponds to this special case of partial area loading [3].

2. State of the art of partially loaded areas in reinforced concrete

2.1. Types of partial area loading

When a concentrated force is applied over a limited area of the surface of a concrete block, the resulting internal compressive stresses spread with increasing distance from the loaded area until they eventually approach a uniform distribution over the entire cross-section of the block. Two basic types of partial area loading can be distinguished, depending on the load dispersion being spatial or plane, respectively [4]:

In the *spatial* case (Fig. 1a), the loaded area is significantly smaller than the concrete block in both the transverse (y) and longitudinal (z) directions, resulting in a bidirectional load dispersion. Practical examples of this case are anchorages of post-tensioning cables and bridge bearings. In the *plane* case, the load is applied over the entire length b_2 of the block such that load dispersion is only possible in the y-direction. Because of their fundamentally different structural response (see Section 2.2), the authors propose to further distinguish here between *partial area loading of plates* (Fig. 1b), where the length of the loaded

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Nomenclature

Coordinates

x	loading direction (axial, vertical)
y	transverse direction
z	longitudinal direction

Geometric data

b_1	length of the loaded area (in z direction)
b_2	block length (in z direction)
d_1	width of the loaded area (in y direction)
d_2	block width (in y direction)
h	block height (in x direction)
$m = \frac{b_1 d_1}{b_2 d_2}$	load concentration ratio

Material properties

f_{c0}	concrete cylinder compressive strength
f_{cc}	multiaxial concrete compressive strength
f_{ct}	concrete tensile strength
f_y	reinforcement yield strength

Actions and effects of actions[1]*:*

N	axial force
$\sigma_{x0} = \frac{N}{b_1 d_1}$	average axial stress in loaded area
$\sigma_{xd} = \frac{N}{b_2 d_2}$	uniform axial stress at the end of discontinuity region
Z_y, Z_z	bursting force in directions y and z
Z_{z3}	tensile force in block shoulders in z direction

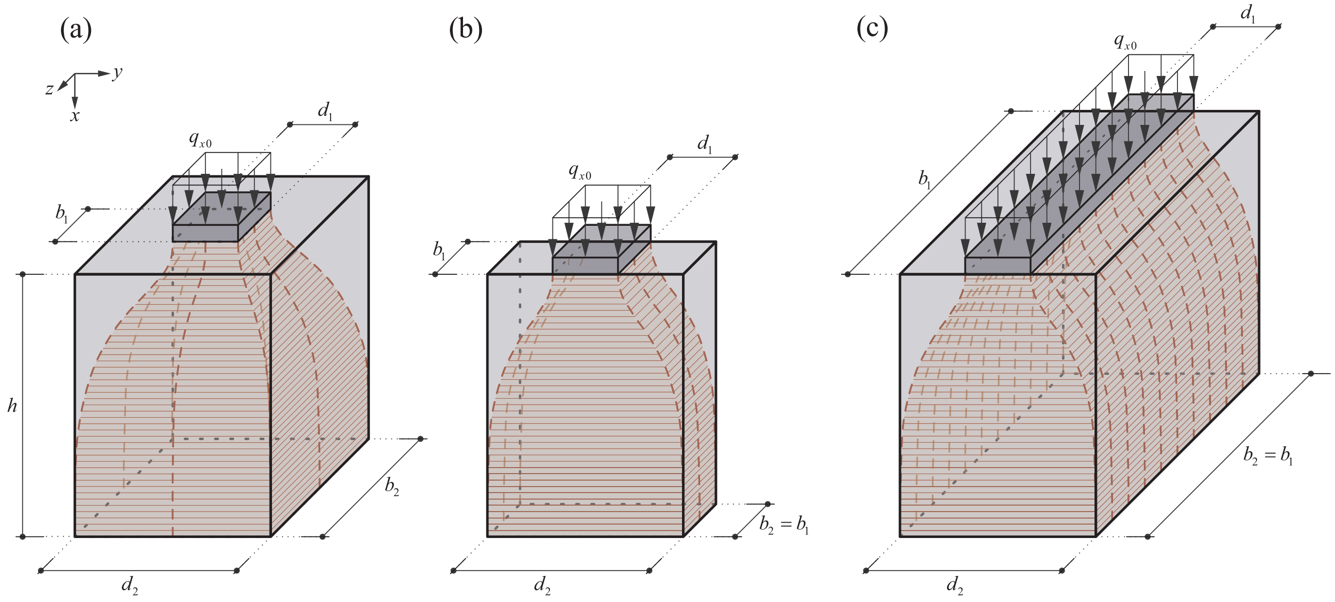

Fig. 1. Cases of partial area loading: (a) Spatial case, (b) plate, and (c) strip loading. Adapted from [4].

area is smaller than roughly twice its width ($b_1 < 2d_1$), and *strip loading* (Fig. 1c), where the loaded area is much longer than wide ($b_1 \geqslant 2d_1$). Practical examples of partially loaded plates are columns supported on walls of equal thickness and vice versa; typical examples for strip loading are one-way concrete hinges and longitudinal joints between segmental tunnel linings.

Cases where the loaded area is much longer than wide ($b_1 \geqslant 2d_1$), but does not extend over the entire length b_2 of the block can be treated as strip loading with good accuracy provided the unloaded length is small compared to the block length, i.e. $b_2 – b_1 \ll b_2$. Numerical simulations [5] show that in such cases, the small stress dispersion in the longitudinal direction merely results in a local confinement near the side faces of the block, which may prevent concrete spalling but does not affect the overall behaviour.

2.2. Mechanical behaviour of partially loaded areas and failure modes

As a consequence of the dispersion of force below the loaded area, bottle-shaped compressive stress trajectories (Fig. 2a) and corresponding deviation stresses perpendicular to these trajectories result within the block. Hence, as illustrated in Fig. 2b, transverse compressive stresses result in the *confined region* immediately below the loaded area, and transverse tensile (bursting) stresses arise in the *bursting region* further away from it, typically starting at a depth approximately equal to the width d_1 of the loaded area. Accordingly, failure under partial area loading can be caused not only by crushing of the concrete below the loaded area, but also by tensile failure of the bursting region, which in fact tends to occur more often. However, these failure modes are difficult to distinguish in experiments since both essentially result in a wedge-shaped (plane case) or pyramidal (spatial case) failure zone below the loaded area, progressively penetrating the concrete block.

2.2.1. Bursting region

Early research on partial area loading focused on the structural behaviour of bursting regions. The magnitude and distribution of the bursting stresses primarily depends on the applied axial compressive stresses $\sigma_{x0} = N/(b_1 d_1)$, the *load concentration ratio* $m = (b_1 d_1)/(b_2 d_2)$ and the reinforcement layout. The bursting stress distribution, illustrated in Fig. 2b for a linear elastic homogeneous block, roughly corresponding to the conditions in uncracked concrete, is greatly affected by the formation of cracks once the tensile strength of concrete is exceeded. Subsequent yielding of the bursting reinforcement results in further significant stress redistributions. After several researchers (e.g. [6–8]) investigated the linear elastic stress distribution of partially