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Dynamic behavior and vibration mitigation of a spatial tensegrity beam

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ABSTRACT

The present study focuses on employing an efficient control strategy to vibration mitigation for a spatial tensegrity beam composing of five typical quadruplex units. The active control approach is implemented in order to maintain the structural integrity and stability under dynamic loadings. Linear-quadratic regulator (LQR) is applied based on various actuator placement scenarios. Five scenarios are carried out for the control strategy, i.e. struts, cables and combination by change actuator positions in different regions. Various actuator placement schemes together with the system response are compared for all simulated scenarios by setting control parameters of LQR. The outcome of the proposed study highlights the significant reduction of dynamical response compared to the uncontrolled performance, succeeding in the tensegrity system control, even at low cost with only 5.26% (4 out of 76) elements controlled.

1. Introduction

Tensegrity structures are spatial, reticulated and lightweight structures that have been known for almost half a century [1,2], and seems to be one of the most promising with positively controlled structures because of its large motion amplitude together with its big strength-to-mass ratio values [2–5]. These smart systems have a large number of practical applications, for example, a small range of transportation, tunable stiffness, active vibration damping and deployment or configuration control [3–5]. As one of the novel structural systems with a wide space application [3], tensegrities have been received significant attentions from quite a large number of researchers [4–7]. The tensile cables and compressed struts with initial forces are the necessary conditions for maintaining the stable configurations of tensegrities. Hence, the structural stiffness that makes the whole tensegrity system flexible depends on the material properties and the initial force conditions [8–11]. Based on the aforementioned existing studies, it is concluded that only small amount of energy is needed to control the shape of tensegrity structures, which is advantageous for active control.

Until now, there are a large number of literatures related to dynamic analysis of tensegrities. Motro et al. [12] performed dynamic experimental and numerical work on a tensegrity structure composed of 3 cables and 9 struts. They showed that a linearized dynamic model around an equilibrium configuration provides a good approximation of the nonlinear behavior of simple tensegrity structure. Ben et al. [13,14] proposed a numerical procedure for nonlinear dynamic analysis of tensegrity systems. Oppenheim and Williams [15] concluded that friction in the rotational joints of the structure is a more important source of damping than the damping in tendons. They also examined the dynamic behavior of a simple elastic tensegrity structure, and found that the natural damping of the tensegrity elements is poorly mobilized due to the existence of infinitesimal mechanisms [16]. Sultan et al. [17] derived linearized dynamic models for two classes of tensegrity structures and proved that the modal dynamic range generally increases with the pretension. Masic and Skelton [18] utilized a linearized dynamic model to enhance the dynamic control performance of a tensegrity structure. Tan and Pellegrino [19] investigated the nonlinear vibration of a cable-stiffened pantographic deployable structure and showed that the system resonant frequencies are related to the level of active cable pretension. The form finding process should take the symmetric geometry relationship of tensegrity and the equilibrium conditions into consideration, as such, there were some researcher who worked on dynamic analysis of different configurations but yields to the same symmetric conditions. For example, Zingoni [20] applied a group-theoretic method to symmetric of grid configurations; Chen and Feng [21] extended the model analysis by adding prestressed loads to them.

Since tensegrity systems can be decorated with active controller, they need to be actively controlled for the sake of structural serviceability and safety. The concept of active control for structure systems were introduced by Yao [22] at the beginning of 70s in 20th century and afterward it was introduced to tensegrity system control by the mid of 90s. A typical tensegrity experiment was implemented by Domer and

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Smith [23]. Chan et al. [24] and Fest et al. [25] presented an active shape-control method to tensegrity structure experimentally. Djouadi et al. [26] described the active vibration control of class 2 tensegrity structure undergoing large deformations by optimal control scheme coupled with a finite element method. Sultan et al. [8] presented a formulation of active control and illustrated it with the example of an aircraft motion simulator. Bel et al. [9] described the dynamic behavior and vibration control of a full-scale active tensegrity structure. The vibration control of a tensegrity beam under sinusoidal excitation is investigated by Nelson [10]. Korkmaz et al. [27] have studied an active control methodology for different cable node positions of a pedestrian bridge with damage scenarios. The comparisons between discontinuous cables controlled schemes to continuous cable controlled schemes have shown some encouraging results for the real application. Veuve et al. [28] presented a two-stage control method for the connection of two halves of a footbridge by computation active cable changes and measuring the response of deployment. Apart from these, some other work also presented the applicability for active control strategy to tensegrities [10,29,30].

However, with the context of structural dynamic control, a number of shortages are reported for the unsatisfied performance control strategies in different implementations. In general, there are two aspects that cause the ineffectiveness: due to the flexible characteristic, they have many vibratory modes, which is a quite challenging task to control their dynamic performance especially in real application because of controller bandwidth limitation [31–33]. For instance, some modes are beyond the range of controller bandwidth; as a typical nonlinear system, tensegrities dynamic outputs influenced by the initial prestressed condition for the cable and strut components, i.e., the force density coefficient has great influence on the global stiffness of the whole system [12,34–37]. For controlling the system operationally, the process has to apply some system identification algorithms to extract model information firstly, then to design the reasonable controllers. Nevertheless, there are quite a few robust algorithms for nonlinear system in terms of vibration mitigation. Thus, it is burdensome to take effective identification results for the control requirements, and even more challenge if system uncertainties have been taken into consideration.

Although many studies obtained results mainly from numerical simulation of small, simple, and symmetric tensegrity models, to the best of the authors' awareness, few work has been reported in the work on the application to complex spatial tensegrity systems. Inspired by these ideas, in this paper, the linear quadratic regulator (LQR) in conjunction with different actuator placement schemes are employed to implement the active control of the spatial tensegrity systems under dynamic excitation. In order to create structures which can adapt to maintain stability [29] and vibration control method level [38–40], the actuators in general are employed to the elements of tensegrities [41,42]. The importance of this study is the effort in realization a relatively simple control strategy applying to relatively complex spatial tensegrity systems, meanwhile, optimal active control is considered by optimizing the structure and controller simultaneously; on the other side, different control scenarios have been compared on their control effects to guide engineering applications.

The rest of content is organized as follows: Section 2 contains the basic assumptions of the geometrical connection conditions for struts and cables of tensegrity system. The other content in the aforementioned section is the problem formulation of spatial tensegrity system, which consist of mass matrix, contribution different parts of the stiffness matrix. Afterward, the optimal active control methodology is accordingly outlined in Section 3, followed by a double layer quadruplex tensegrity beam as a numerical application. Lastly, in the closing section, the simulation results are summarized and critical comments are made for further research work planned to be incorporated.

2. Dynamic model of spatial tensegrity system

2.1. Basic assumptions

Although tensegrity systems show highly nonlinear dynamic performance [42–46], for the simplification reason, the linear control method around an elastic equilibrium offers a well approximation [31]. Before establishing dynamic model of the spatial tensegrity system, the following assumptions are employed [2,37]:

- Cables and struts are connected by pin joins;
- Strut members carry axial tensile or compressive forces;
- Cable members only carry axial tensile forces;
- The external loads only act on the nodes of whole system;
- The local and global bucking of struts are neglected;

For whole cables and struts, the dynamic incremental stiffness caused by external loads is much smaller compared to their linear and geometrical stiffness such that incremental stiffness can be ignored.

2.2. Problem formulation

In this section, a linearized dynamic model around an equilibrium configuration is used to describe the dynamic behavior of the active tensegrity system under harmonic excitation. The linearized differential equation at a prestressed configuration can be written as:

$$\hat{\mathbf{M}}_b\ddot{\hat{\mathbf{x}}}(t) + \hat{\mathbf{C}}_b\dot{\hat{\mathbf{x}}}(t) + \hat{\mathbf{K}}_b\hat{\mathbf{x}}(t) = \mathbf{A}_0 sin(\varpi t + \vartheta) = \hat{\mathbf{F}}(t) \tag{1}$$

Here, $\hat{\mathbf{M}}_b$, $\hat{\mathbf{C}}_b$ and $\hat{\mathbf{K}}_b$ refer to the mass, damping and stiffness matrix, respectively, $\hat{\mathbf{F}}(t) = [\hat{F}_i(t)]$ represents the excitation inputs matrix, vector $\hat{\mathbf{x}}(t)$, $\dot{\hat{\mathbf{x}}}(t)$ and $\ddot{\hat{\mathbf{x}}}(t)$ represents the vector of nodal displacement, velocity and acceleration respectively, $\mathbf{A}_0$ refers to the dynamic loading amplitude coefficient matrix, ϖ is the frequency of dynamic loading and ϑ is the phase angle. For the development of a finite element model of the tensegrity system, each element in the structure is characterized as [42]:

$$\hat{\mathbf{M}}_b^e = \left(\frac{m_{ij}}{6}\right)\begin{bmatrix} 2\mathbf{I}_3 & -\mathbf{I}_3 \\ -\mathbf{I}_3 & 2\mathbf{I}_3 \end{bmatrix} \tag{2}$$

where is the element mass matrix, $\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and m_{ij} represents the mass contributes by element*ij*.

On the other hand, Rayleigh damping model is considered in this paper to compute the damping matrix$\hat{\mathbf{C}}_b$. Rayleigh defined proportional damping as a dissipative situation where viscous damping is directly proportional to mass, stiffness or both as Eq. (3): here α_c and β_c represent mass and stiffness material loss factors respectively, which can be determined by solving the nature frequencies and damping ratios of the first and second modes of the structure system.

$$\hat{\mathbf{C}}_b = \alpha_c\hat{\mathbf{M}}_b + \beta_c\hat{\mathbf{K}}_b \tag{3}$$

A space rod element for initial and current configuration is illustrated in Fig. 1. The element nodal coordinates at initial time and time t in the global coordinate system XYZ is given by: (x_i^0,y_i^0,z_i^0), (x_j^0,y_j^0,z_j^0), (x_i^t,y_i^t,z_i^t), (x_j^t,y_j^t,z_j^t), correspondingly. Additionally, the global displacement vector is computed by

$$\mathbf{d} = [\mathbf{u}_i\ \mathbf{v}_i\ \mathbf{w}_i\ \mathbf{u}_j\ \mathbf{v}_j\ \mathbf{w}_j]^T = [x_i^t - x_i^0, y_i^t - y_i^0, z_i^t - z_i^0, x_j^t - x_j^0, y_j^t - y_j^0, z_j^t - z_j^0]^T \tag{4}$$

The local displacement along the element is as follows

$$\Delta l = l^t - l^0 \tag{5}$$

where l^t and l^0 are the length of the member at time t and initial time, respectively, given as: