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Response spectrum method for building structures with viscoelastic dampers described by fractional derivatives

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ABSTRACT

This study concerns the structural systems in which additional damping was introduced by means of viscoelastic (VE) dampers modeled with the fractional derivative models. The fractional derivative models have an ability to correctly describe the behavior of VE materials in a wide range of frequency, using a small number of model parameters. However, the governing equation of motion includes fractional derivatives together with ordinary ones. In the proposed approach, after applying the Laplace transform and the inverse transform to the equations of motion, the solution obtained for the system with fractional dampers is equivalent to the modal solution used in the case of proportional damping. In order to validate the proposed approach, the maximum response of the structural system equipped with dampers is determined in the time domain. Thus, the equations of motion with fractional derivatives derived for the considered system are numerically integrated. Moreover, the paper extends the response spectrum concept to structures equipped with fractional viscoelastic dampers.

1. Introduction

The time integration method and the response spectrum method are often used in the dynamic analysis of structures subjected to earthquake forces. The time integration method is used in the time domain while the spectrum method is used in performing the analysis in the frequency domain. Real structures are systems with many degrees of freedom and, in this case, the analysis in the time domain could be very time consuming. Therefore, the spectrum method is recommended in design codes in many countries. In the spectrum method, the system with many degrees of freedom is transformed to a set of systems with one degree of freedom in the modal subspace. The maximum response of structure to seismic forces can be obtained by the superposition of maximal responses of the above-mentioned set of one degree of freedom oscillators. For classically damped linear systems with proportional damping matrices, the linear modes of vibration are used for uncoupling the equation of motion [1,2]. However, for a non-classically damped system, such as structures with viscous or viscoelastic dampers, the classical response spectrum method cannot be used because of the non-proportionality of the damping matrix. In order to uncouple the equation of motion of structures with non-proportional damping, matrix properties of the complex eigenvalue problems must be used [3]. In the end, a set of first-order uncoupled differential equations with complex coefficients are obtained. In this context, various research papers were published by Velesos and Ventura [4], Yang et al. [5], Maldonado and Singh [6], Falsone and Muscolino [7,8], Zhu et al. [9] and Liu et al. [10], among other ones. In the paper [11], the spectrum method was used for the dynamic analysis of structures with viscoelastic dampers modeled with the help of the generalized Kelvin and Maxwell rheological models while in [12], the spectrum method was applied in the analysis of structures with active control systems.

Structures subjected to dynamic loads caused by earthquakes or wind pressure could be equipped with different types of damping systems in order to reduce excessive vibrations. Depending on whether the energy is provided to the damping systems, they are divided into active, passive and semi-active systems. Different kinds of passive systems are often used to reduce excessive vibrations because these types of systems are effective, simple to manufacture and no external source of energy is required for their operation [13,14]. The viscoelastic (VE) dampers are often used with success as passive control systems [11,15–19], among other ones. The viscoelastic dampers are built of polymeric materials which are able to dissipate vibration energy [20]. These dampers mounted in structures added damping as well stiffness. Moreover, the properties of viscoelastic materials depend on the frequency of excitation, environmental temperature and amplitudes of vibration when such amplitudes are large [20]. The dynamic behavior of VE dampers could be described by rheological models which could be either classical or so-called fractional derivative ones. The classical rheological models are used in [11,18,21,22] while the fractional derivative models are adopted in [15–19,23]. In contrast to the classic rheological models,

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Nomenclature

$a_{gr}(t)$	ground acceleration
c_i	damping parameter of damper model
k_i	stiffness parameter of damper model
$q_i(t)$	degree of freedom of damper or structure
s	Laplace variable
s_i	complex and conjugate eigenvalue
$u_0(t)$	force in Kelvin model of damper
$u_1(t)$	force in Maxwell model of damper
$E[\bullet]$	expected value of random variable
D_i, E_i	peak values taken from spectra
D_t^α	derivative of order α with respect to time t
L^{-1}	inverse Laplace transform
$\mathbf{a}_i$	eigenvector with complex components
$\mathbf{e}_r$	vector of damper location
$\mathbf{f}(t)$	vector of interaction forces between structure and dampers
$\mathbf{i}$	location vector of inertia forces in structure
$\mathbf{p}(t)$	vector of excitation forces
$\mathbf{q}(t)$	vector of structure displacements
$\mathbf{x}_j$	real part of eigenvector
$\mathbf{z}_j$	imaginary part of eigenvector
$\mathbf{C}_d$	damping matrix of dampers
$\mathbf{C}_s$	damping matrix of structure
$\mathbf{D}(s)$	dynamic stiffness matrix
$\mathbf{G}_d$	matrix combining stiffness and damping properties of dampers
$\mathbf{H}(s)$	frequency response matrix
$\mathbf{K}_d$	stiffness matrix of dampers
$\mathbf{K}_s$	stiffness matrix of structure
$\mathbf{L}_r$	location matrix of damper
$\mathbf{M}_s$	mass matrix of structure
$\mathbf{R}_j$	matrix of residues
α	order of fractional derivative
γ_i	non-dimensional damping ratio
η_i	imaginary part of eigenvalue
λ	frequency of harmonic vibration
μ_i	real part of eigenvalue
ν_1	relation between stiffness and damping parameters in Maxwell model
$\rho_{i,jk}$	cross-correlation coefficient
ω_i	natural frequency of vibrations
ω_{jD}	damped angular frequency

which require a large number of parameters to correctly describe the behavior of dampers [18,21], the fractional derivative models have an ability to correctly describe the behavior of VE materials in a wide range of frequency using a small number of model parameters [18]. The complex modulus model [20] is also often used but the dynamic analysis must be done in the frequency domain.

Only Chang and Singh in [15] and Singh and Chang in [11] gave systematic analyses of the fractional derivative Kelvin model of VE dampers in the context of seismically excited vibration of structures. The fractional derivative model is restricted to the case when the order of fractional derivative α can be written as a quotient of two integer numbers. The response spectrum concept was not presented in detail but some response spectra of building floors are shown. Moreover, a very large linear eigenvalue problem must be solved before the derivation of the set of uncoupled modal equations. In papers [11,15–17], the seismic, in time domain analyses of structures with fractional dampers are also presented.

The aim of this paper is to determine the dynamic response of a building structural system equipped with viscoelastic dampers and subjected to seismic loads. The dynamic behavior of dampers is described by a set of rheological models with both classic and fractional order derivative. The dynamic analysis of the considered system is carried out in the frequency domain. The paper extends the response spectrum concept to structures equipped with fractional viscoelastic dampers. The cross-correlation coefficients are determined for the complex-valued mode shapes obtained in the case of non-classically damped systems. These coefficients enable determination of the peak values of displacements and internal forces for structures, subjected to the ground acceleration. The remaining part of the paper is organized as follows. In Section 2, the adopted rheological models of VE dampers together with the equation of motion of structures with VE dampers are presented. The solution in the frequency domain of the motion equation for structures with dampers is described in Section 3, while in Section 4 the solution in the time domain is given. Extension of the spectrum response concept for the structures with dampers modeled with the help of fractional derivatives is proposed in Section 5. The results of typical calculations are presented and discussed in Section 6. The paper ends with concluding remarks and Appendix A.

2. Non-classically damped structures

2.1. Rheological damper model

The dynamic behaviors of the VE dampers are typically described using rheological models (e.g. Kelvin model, Maxwell model). The rheological damper model usually includes a number of suitably combined springs and dashpots [14,21]. In general, the behaviors of conventional viscoelastic materials depend on frequency, temperature and deformation amplitude. However, in order to adequately describe the properties of the material, a large number of coefficients must be used. The model has the disadvantage of a significant difficulty in identifying all of the coefficients for a real material.

The rheological properties of VE dampers could also be described using the fractional calculus (see [15,24]), i.e., fractional derivatives in the description of a mechanical model. Fractional differentiation is the operator that generalizes the order of differentiation to fractional

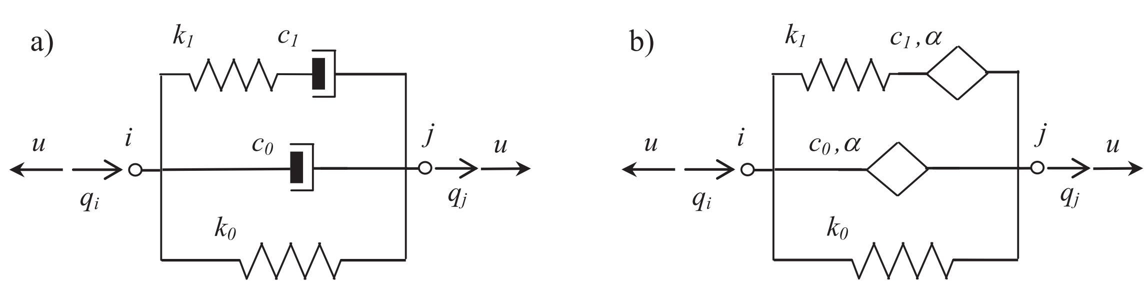

Fig. 1. Complex model of damper: (a) classical and (b) fractional.