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Three-parameter viscoelastic foundation model of adhesively bonded single-lap joints with functionally graded adherends

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ABSTRACT

Functionally graded (FG) adherends are beneficial to adhesively bonded joints, as different material composition through the adherend thickness can manipulate the interfacial stress distribution. It is a fact that both shear and peeling stresses along the adhesive/adherend interface at the free edges play important roles in the structural integrity of a joint. Previous studies have shown that both shear and peeling stresses can be uniformly distributed near the edges of a joint with FG adherends if the right adherend composition is selected through the adherends' thickness. However, the effect of the viscoelastic behavior of the adhesive layer has been neglected in those studies. This study establishes a viscoelastic analytical model for adhesively bonded single-lap joints with FG adherends. In this model, the adhesive layer is simulated as a three-parameter viscoelastic foundation using a standard linear solid (SLS) model to account for both creep and relaxation behaviors in the adhesive. This model satisfies the zero-shear-stress boundary at the free edges of the adhesive and predicts different peel stresses along two adherend/adhesive interfaces. Excellent agreement with finite element analysis (FEA) has been achieved by the present model, confirming the accuracy of the model. The viscoelastic behavior of the adhesive layer affects stress concentration near the edges of a joint at early ages, suggesting that the present model can properly capture the stress relaxation in adhesively bonded joints with FG adherends. The parametric studies show that FG material configuration and the mechanical properties of adhesive layers play an important role in the uniformity of shear stress distribution along the length of single-lap joints with FG adherends. The present viscoelastic solution can predict more uniform stress distribution and is a valuable tool in design optimization of joints with FG adherends.

1. Introduction

Adhesively bonded joints are widely used in composite structures to connect components due to their many advantages compared with other joining methods. However, premature failure due to debonding and peeling of joints is the major concern with this technique. To address this issue, many analytical and numerical solutions have been proposed to precisely evaluate the stresses in adhesively bonded joints. Among them, the most successful analytical model was proposed by Goland and Reissner (G-R model) [1], in which the adhesive layer is modeled as a series of uniformly distributed shear and vertical springs independent of interaction with each other (For abbreviations, see Appendix B, Table B1). The G-R model is essentially a two-parameter elastic foundation model (2PEF), which only considers two parameters in the adhesive layer: shear stress and a single peel stress. Based on the G-R model, a number of models have been developed for various problems [2,3,12–14,4–11]. However, all these models ignore the zero-shear-stress condition at the free ends of the adhesive layer. To overcome this shortcoming, Wang and Zhang proposed a new model for the adhesive layer. Unlike in the G-R model, which assumes that the peel stress along any two adherend/adhesive interfaces is the same, Wang and Zhang's model removes this assumption and considers these two peel stresses as different [15]. As a result, three stresses (two peel stresses and one shear stress) exist in an adhesive layer. To relate these three stresses to the deformation of the adhesive layer, Wang and Zhang [15] model the adhesive layer as two layers of peel springs connected through a layer of shear strings, which is essentially same as a three-parameter elastic foundation. Therefore, Wang and Zhang's model [15] is referred to as three-parameter elastic foundation (3PEF), which can be viewed as a direct extension of the two-parameter elastic foundation model used in the G-R model. By adopting this 3PEF model, the zero-shear-stress boundary condition at the free edge of the adhesive layer can be satisfied. More importantly, different peel stresses along two adherend/adhesive interfaces are produced by this model, which can

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predict more precisely where debonding will initiate.

Functionally graded materials (FGM) are new inhomogeneous composite materials which are increasingly being used in aerospace industries and as aeronautical components to resist elevated temperature. For example, these materials are used as components in aircraft, space rockets, launchers and space shuttles [16]. The concept of tailoring material composition was initially proposed for composite and polymeric materials in the early 1970s [17,18]. However, it was not until around the mid-1980s that a group of Japanese scientists proposed the manufacture, design and analysis of FGMs [16]. The idea received special attention, and after a few years the first international symposium on FGMs was held in Tokyo, Japan [19]. FGMs are made of either uniformly or nonuniformly distributed materials with different mechanical properties, sizes or shapes. Combinations of ceramic and metal, by and large, are used for FGM in the aviation and aerospace industries because of the many thermal and mechanical behaviors they produce [20]. Many researchers have studied the thermal and mechanical properties of FGM plates resting on elastic foundations in recent years [21–25]. Zidi et al. studied the bending response of functionally graded plates via a four-variable refined plate theory [22]. The stress and displacement responses were analyzed under hydro–thermo-mechanical loadings on an elastic foundation modeled as a two-parameter Pasternak foundation. It was shown that moisture concentration has a significant effect on the results. Meziane et al. [23] developed a model based on shear deformation theory in order to simulate the vibration and buckling of exponentially graded material sandwich plate resting on elastic foundations.

It should be noted that many natural biomaterials such as teeth, bones, bamboo and seashells have functionally graded properties [19,26]. Engineered FGMs, similar to natural functionally graded structures, provide uniform interfacial stress distributions and reduce stress concentrations [19,27,28]. Guin and Wang [29] proposed a three-parameter model that could obtain a closed-form expression for the interface stresses of functionally graded (FG) adherends. Nevertheless, this model only focused on the elastic foundation model of the adhesive layer, so the viscoelastic behavior of the adhesive joint was neglected.

Structural adhesives such as epoxy exhibit viscoelastic properties. These material properties vary with time under different situations, especially in the regions of high-stress concentration [30–33]. Such variation of material properties can induce redistributions of stresses and additional deformation, which could be significant during the service life of the structure and cause potential failure of its joints. Therefore, some efforts have been made to study the viscoelastic behaviors of adhesive joints [34,35,44–50,36–43]. It has been shown by most of these studies that the ratio of shear stress through the adhesive layer to shear strength is an important contributor to the viscoelastic behavior of adhesively bonded joints [2]. In earlier studies, assumptions such as quasi-elastic approximations were made to simulate bulk material. In this way, the viscoelastic response of the structures could be approximated as an elastic solution in which the elastic constants are replaced by time-dependent creep or relaxation functions of the materials [51]. Delale and Erdogan developed a viscoelastic solution for symmetric adhesive joints [36]. Mirman and Knecht proposed another, more simplified viscoelastic model by ignoring the peeling stress of the adhesive layer [31]. However, this model assumes both the shear and peel stresses are constant through the thickness of the adhesive layer, which introduces three inherent flaws to the model: (1) the zero-shear-stress boundary condition at the free edge of the adhesive layer is not satisfied; (2) the equilibrium equation of the adhesive layer is not satisfied; and (3) the different peel stress distributions along two adherend/adhesive interfaces are not distinguishable. A viscoelastic solution capable of predicting both the shear and peeling stresses along the interface was proposed by Zhang and Wang for FRP-strengthened reinforced concrete beams [52]. Although this model can closely simulate the viscoelastic behavior of the FRP-strengthened beams, it is only applicable to isotropic adherends.

Most existing studies have adopted numerical finite element methods in order to simulate the time-dependent behaviors of adhesively bonded joints. These finite element models are mostly based on the rheological models obtained from experiments on creep behaviors of adhesive layers [38,46–48,52]. Although experimental tests and numerical methods are the most accurate methods in parametric studies of the time-dependent behaviors of the adhesive joints, there are some drawbacks to these procedures: first of all, both experimental and numerical methods are generally time-consuming. It may take days or months to conduct an experimental test on the viscoelastic behavior of a structure. By the same token, numerical models will take a long time to complete analysis, as smaller iteration steps are required to avoid error accumulation [52]. Furthermore, the complexity of finite element simulations and the excessive cost of experimental studies make them less attractive to engineers. For these reasons, closed-form analytical solutions are most desirable. To date, no rigorous closed-from solution for time-dependent behavior of adhesively bonded joints with FG adherends exists. The major objective of the present study is to address the abovementioned gaps. This study proposes a viscoelastic analytical solution to the pre-crack interface stresses of joints with FG adherends. We propose a three-parameter viscoelastic foundation model (3PVF) to simulate time-dependent stress distribution in symmetric, adhesively bonded joints with FG. The present model is capable of capturing the critical long-term stress distribution of these joints at free edges. Our new model is built on the model of Zhang and Wang [52] by considering the viscoelastic behavior of the adhesive layer for adhesively bonded single-lap joints with functionally graded adherends. Not only does this study provide an analytical solution based on a three-parameter viscoelastic foundation model that can capture different peel stresses along adherend/adhesive interfaces, but it also defines the uniformity of the shear stress distribution and its role in the effectiveness of adhesively bonded joints with FG adherends.

2. Three-parameter viscoelastic foundation model (3PVF) for a symmetric single-lap joint with FG adherend

A general single-lap joint with applied loads (shown in Fig. 1) is considered. This joint may be symmetric, in which two adherends are identical, or asymmetric, in which the two adherends are different. As shown in Fig. 1, the joint consists of three different layers: two adherends with thicknesses of h_1 and h_2, respectively, and one adhesive layer with thickness of h_0. (A list of nomenclature is given in Appendix B, Table B2.) The free-body diagram of the overlap area of the joint is shown in Fig. 2. Modeling two adherends as Timoshenko's beams, we have

$$U_i(x_i,z_i,t) = u_i(x_i,t) + z_i\phi_i(x_i,t), W_i(x_i,z_i,t) = w_i(x_i,t), \quad (i = 1,2) \tag{1}$$

where subscripts (i = 1, 2) represent the top and bottom adherends, respectively. $u_i(x_i, t)$, $\phi_i(x_i, t)$ and $w_i(x_i, t)$ are the axial displacement, the rotation and the deflection at the neutral axis of the beam i, respectively. Since $x_1 = x_2 = x$, subscript i (i = 1, 2) in displacement

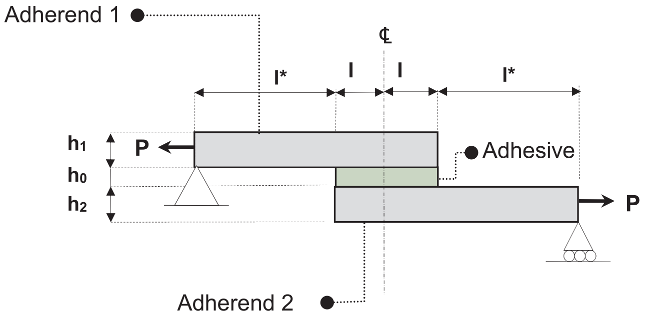

Fig. 1. A simply supported single-lap joint.