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Nonlinear finite element simulation of unbonded prestressed concrete beams

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ABSTRACT

Prestressed concrete with internal unbonded tendons has been recognized as an excellent structural option for beams and slabs and is employed worlwide. Numerical solutions for the analysis of such structures are still an active field of research. This work presents a finite element model for the physical and geometrical nonlinear analysis of prestressed concrete beams with unbonded internal tendons, under short-term loading. The reinforced concrete beam is modeled by Euler-Bernoulli nonlinear plane frame elements and a total Lagrangian approach. The prestressing tendon is modeled by a single polygonal element embedded in a specified subset of the frame elements. Due to lack of strain compatibility between the concrete and the tendon at a given cross-section, the cable strain is computed from the displacements of all associated frame elements. Geometric and material nonlinearities are considered for both the reinforced concrete beam and the prestressing tendons. The internal force vector and corresponding tangent stiffness matrix of each element under large displacements are derived consistently, and novel expressions for the tangent stiffness operator which ensure the convergence rates of the Newton-Raphson scheme are developed. The accuracy of the formulation is assessed by comparison with experimental tests, with very good results.

1. Introduction and literature overview

Prestressed concrete structures are mainly divided into two groups, according to the application of the prestressing forces to the concrete element. Pretensioning considers that the steel is tensioned before the concrete is cast, therefore requiring bond between the elements for the proper force transfer. Post-tensioning, on the other hand, implies that the tendon or cable applies stresses upon the concrete element already during the prestressing operation. In this case the bond between prestressing steel and concrete depends on the constructive solution adopted.

Unbonded prestressed concrete structures are a very efficient load-carrying system, especially with the use of greased low-cost tendons protected by plastic sheathing. These elements have been used extensively in North America for over 50 years [1] and have become popular in the construction of medium rise buildings in Brazil in the last decades.

The numerical simulation of these structures is a challenging task, as one must necessarily cope with the first stages of application of tensioning, the immediate loss of prestress, the behavior under external loads and, in case of a long-term analysis, the loss of prestress due to phenomena such as shrinkage, creep and tendon relaxation.

Numerical analysis of unbonded prestressed concrete beams tends to be more complex than that of the bonded case, since in the former there is no strain compatibility between the concrete and the tendon at a given cross-section. Thus, the cable strain depends on the displacements of the tendon as a whole.

The crucial issue, therefore, is the consideration of the slipping tendon. The most employed strategy has its roots on the load balancing concept, which was originally introduced by Lin and Burns [2] and afterwards extended by Aalami [3,4]. According to this approach, the tendon acts as an external force applied to the concrete beam.

The FE simulation of mechanical behavior of prestressed concrete beams with unbonded tendons is still the subject of various research works. In the following some of the most recent ones, more closely related to the present work, are briefly described.

Barbieri et al. [5] developed an hybrid FE model for bonded and unbonded prestressed concrete frames, where the active and passive reinforcements are modeled as layers within the cross section. The bonded tendon contributes to the overall stiffness, but the unbonded tendon is considered as an equivalent force which does not contribute to the stiffness coefficients.

D'Allasta and Zona [6] developed a FE model for externally prestressed composite beams with deformable connection. Later the same research group presented a formulation for nonlinear analysis of beams prestressed with external slipping tendons [7], as well as analytical and

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simplified procedures for prestressed beams [8–10] Their FE model takes into account the influence of the tendon change of position in the tangent stiffness matrix in the context of a high order displacement-based FE formulation.

Lou and Xiang [11,12] developed a numerical procedure to investigate second-order effects on externally prestressed concrete beams. Later, Lou et al. [13,14] included the effect of long-term behavior on the nonlinear analysis of prestressed concrete girders within a similar nonlinear FE formulation. The same authors have also applied their numerical formulation for beams prestressed with FRP tendons [15,16], for prestressed concrete columns [17] and externally prestressed steel-concrete composite beams [18].

Vu and coworkers [19] developed a nonlinear FE model for the structural response of post-tensioned beams, based on a so-called macro finite element which is characterized by its homogeneous average inertia. Kim and Lee [20] developed a flexural analytical model focused on the behaviour of continuous unbonded post-tensioned members.

The present work focuses on the analysis of prestressed concrete beams with unbonded internal tendons, under short-term loading. The reinforced concrete beam is modeled by nonlinear plane frame elements based on the Euler-Bernoulli-Navier beam theory and the total Lagrangian approach. Each unbonded tendon is modeled by a single polygonal element embedded in the correspondent the frame element. Geometric and material nonlinearities are considered for both the reinforced concrete beam and the prestressing tendons. The internal force vector and tangent stiffness matrix of each element under large displacements are derived in a variationally consistent way, providing optimal rates of convergence to the nonlinear analysis procedure Special attention is given to the development of consistent and robust numerical approaches for the two stages of short-term loaded prestressed concrete beams: tendon stressing and further load application. The accuracy of the formulation is assessed by comparison with experimental results.

2. Frame element

The proposed model employs an Euler-Bernoulli displacement-based frame element for the simulation of the reinforced concrete member. The hypotheses of plane sections, with large displacements and moderate rotations, widely used on analysis of reinforced concrete frames, are considered in the context of a Total Lagrangian description. The displacement field can be written as:

$$u(X,Y) = u_0(X) - Y\, v_0'(X) \quad v(X,Y) = v_0(X) \tag{1}$$

where u and v are the axial and transverse displacements and the subscript 0 refers to displacements at the beam reference axis. The element geometry and coordinate system are shown in Fig. 1. The analytical model considers the membrane ε_0 and curvature κ terms, with the geometric nonlinearity taken into account in the membrane strain as follows:

$$\varepsilon = u_0' + \frac{1}{2}v_0'^2 - Yv'' = \varepsilon_0 - Y\,\kappa \tag{2}$$

The membrane strain and the curvature can be interpreted as generalized strains and written in vector form as:

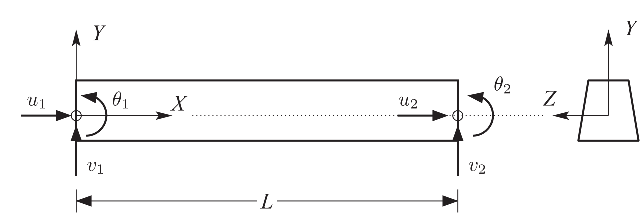

Fig. 1. Frame element geometry and degrees of freedom.

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_0 \\ \kappa \end{bmatrix} = \begin{bmatrix} u_0' \\ v_0'' \end{bmatrix} + \frac{1}{2}\begin{bmatrix} v_0'^2 \\ 0 \end{bmatrix} \tag{3}$$

where the first and second vector represents the linear (ε_L) and non-linear (ε_{NL}) generalized strains.

The axial stresses (σ) can be computed from the axial strains (ε) using the constitutive models discussed in Section 4, while the normal force (N) and bending moment (M), henceforth called generalized stresses, are computed from the integration of stresses in the cross-section:

$$\boldsymbol{\sigma} = \begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} \int_A \sigma\, \mathrm{d}A \\ -\int_A Y\,\sigma\, \mathrm{d}A \end{bmatrix} \tag{4}$$

The element degrees of freedom (DOFs) are depicted in Fig. 1. The usual beam minimum continuity requirements imply the use of a C^0 linear and C^1 cubic hermitian interpolation functions for the axial and transverse displacements, respectively. Let $L_i(x)$ be the two axial linear interpolants and $H_i(x)$ be the four transverse interpolant functions. The introduction of these functions allows the representation of the generalized strains from the nodal displacements via the strain-displacement matrices

$$\boldsymbol{\varepsilon} = \mathbf{B}\,\mathbf{u}_e = (\mathbf{B}_L + \frac{1}{2}\mathbf{B}_{NL})\mathbf{u}_e \tag{5}$$

where $\mathbf{B}_L$ and $\mathbf{B}_{NL}$ are respectively the linear and nonlinear strain-displacement matrices and $\mathbf{u}_e$ is the nodal displacement vector of the element. Matrix $\mathbf{B}_L$ contains derivatives of the interpolation functions L_i and H_i and is the same for linear beam elements, as may be seen in any textbook on linear finite elements. For this reason the focus will be on the nonlinear strain-displacement term. The source for nonlinearity is the term $v_0'^2$.

From the displacement interpolation, the term v_0', equal to the cross-section rotation (θ) can be written as:

$$v_0' = \theta = [0 \quad H_1' \quad H_2' \quad 0 \quad H_3' \quad H_4']\mathbf{u}_e = \mathbf{G}\,\mathbf{u}_e \tag{6}$$

Thus, the nonlinear part of the membrane strain is given by:

$$\varepsilon_{NL} = \frac{1}{2}v_0'^2 = \frac{1}{2}\mathbf{u}_e^T\mathbf{G}^T\mathbf{G}\,\mathbf{u}_e \tag{7}$$

This expression contains polynomials terms n x up to quartic, while ε_L is constant along the element. The unbalanced higher order terms can lead to membrane locking due to inability to represent membrane strains associated with inextensional bending [21]. In order to avoid membrane locking, a higher-order interpolation for the axial displacements was adopted in [7], increasing the number of degrees of freedom and the element complexity. A simpler approach is adopted here, whereby the average strain [21] is used instead of the original expression: Thus:

$$\varepsilon_{NL} = \frac{1}{2}\frac{1}{L}\int_0^L v_0'^2\, dX = \frac{1}{2}\mathbf{u}_e^T\mathbf{A}\,\mathbf{u}_e \tag{8}$$

where

$$\mathbf{A} = \frac{1}{L}\int_0^L \mathbf{G}^T\mathbf{G}\, dX \tag{9}$$

Matrix $\mathbf{A}$ is symmetric, constant and can be evaluated analytically, producing a nonlinear strain vector

$$\boldsymbol{\varepsilon}_{NL} = \frac{1}{2}\begin{bmatrix} \mathbf{u}_e^T\mathbf{A} \\ \mathbf{0} \end{bmatrix}\mathbf{u}_e = \mathbf{B}_{NL}\mathbf{u}_e \tag{10}$$

where the implemented form of $\mathbf{B}_{NL}$ is shown. Incrementally, with a view to virtual work application, it can be shown that the following holds:

$$\delta\boldsymbol{\varepsilon} = (\mathbf{B}_L + \mathbf{B}_{NL})\delta\mathbf{u}_e = \overline{\mathbf{B}}\delta\mathbf{u}_e \tag{11}$$

The element internal force vector ($\mathbf{g}_e$) can be obtained from the