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Free and forced vibration analysis of arbitrarily supported rectangular plate systems with attachments and openings

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ABSTRACT

Rectangular plates with different kinds of attachments (continuous or lumped) and various opening shapes are main constitutive parts of almost all engineering structures, e.g. aircrafts, bridges, buildings, ships, offshore structures, etc. Therefore, an assessment of their free and forced responses is generally very important for safe and rational structural design. In this paper, a range of different vibration problems inherent to rectangular plate systems, that can be solved by the energy based assumed mode method is considered. The concept of assumed mode method is outlined together with application of the mode superposition method to forced response calculation for plates under concentrated harmonic forces or enforced boundary displacement. Furthermore, complete mathematical model for vibration analysis of plate structures carrying arbitrary number of spring-mass systems is developed, based on the receptance method application. The plate is modelled by the Mindlin thick plate theory and Timoshenko beam theory is applied for stiffeners. The eigenvalue problem represented with a multi-degree-of-freedom system equation is formulated by Lagrange's equation of motion, while characteristic orthogonal polynomials having the properties of Timoshenko beam functions and satisfying the specified edge constraints are used as approximation functions. The corresponding in-house software is developed and dynamic responses of rectangular plate systems having different sets of edge constraints are analysed. Comparisons of the results with existing solutions and general finite element (FE) software are included, and very good agreement is achieved.

1. Introduction

Rectangular plates are primary constitutive elements in almost all engineering branches: aerospace, civil, mechanical, naval, nuclear, and offshore. The most important engineering problems encountered with plates can be classified into three main groups: bending, stability and vibration [1]. In many practical applications one can find rectangular plates with various opening shapes or reinforced with a number of stiffeners having different cross-section shapes, dimensions and orientations, to increase their loading capacities. Openings are usually present for weight saving, venting, providing accessibility to other parts of the structures or even to alter plate natural frequencies [2]. Also, above mentioned structural elements sometimes carry multiple mass attachments and can be locally supported with concentrated members (like for instance pillars or elastic springs). Furthermore, there can be the necessity to analyse vibration of plates with elastically mounted equipment, which can be modelled as spring-mass systems. Bearing in mind various combinations of boundary conditions it is obvious that vibration analysis of such plate systems becomes rather complicated task.

During the last decades, the finite element method (FEM) has become the most powerful tool for the structural strength and vibration analysis, widely used in practical engineering [3]. It is applied to very complex structures as well as in determination of static and dynamic response of different simple beam-like and plate-like members, and various finite elements are developed and incorporated in general FE software. However, among other drawbacks, existing FE software still mainly require rather lengthy model preparation, and modifications of the models when different topologies are being investigated are rather time-consuming. Therefore, at the preliminary design stage when the structural principal dimensions are being selected, it is useful for engineers to have some simple, fast and reliable computational tool at

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hand. Since analytical solutions are very difficult to derive (and in some cases impossible), the energy based methods with a sort of assumptions and simplifications seem to be a good compromise for preliminary design stage. Then, the final structural assessment can be confirmed by the finite element method.

The state-of-the art in vibration analysis (both free and forced) of rectangular bare plates and stiffened panels with and without openings, respectively, is already presented by the authors in [4–8]. Also, literature on vibration of plate structures carrying rigidly mounted lumped attachments (mass or stiffness) is recently reviewed in [9]. Therefore, in this paper an extensive literature survey on above topics is omitted and only most important information are outlined. Here, some more light was shed on vibration analysis of plate structures carrying arbitrary number of spring-mass systems, which is one of novelties introduced in the general mathematical model.

Although the thick plate theory represents an issue for very long time, due its complexity caused by transverse shear and rotary inertia, it is still a challenging problem. Many concepts, based both on analytical and numerical solution of equilibrium equations, have been worked out [10–13]. There are different analytical methods limited only to simply supported plates or plates with two opposite edges simply supported. They differ depending on which functions are kept as fundamental ones in the reduction of the system of differential equations of motion. For the vibration analysis of plates with arbitrary boundary conditions, including also elastically restrained edges, different variants of the Rayleigh-Ritz (energy) method are on disposal. Accuracy of such methods is dependent on the chosen set of orthogonal functions for the assumed natural modes, where two dimensional polynomials or static Timoshenko beam deflection functions, [14,15] and [16], respectively, can be used.

As indicated above, an overview of methods for the vibration analysis of plates with openings is presented in [4] where the advantages and drawbacks of different ones, for instance the finite difference method [17], the Rayleigh-Ritz [18] and the optimized Rayleigh-Ritz method [19], FEM [20], etc., are discussed.

In the case of stiffened panels, most of the references are related to their static analysis, but a rather limited number to the dynamic one [5]. The most important publications to this problem are reviewed in [6]. Generally, according to [6,21] the most common methods applied to the vibration analysis of stiffened plates can be classified into closed-bound solutions [22], energy methods [23–25] and other numerical methods [21,26,27].

In the case of the vibration analysis of stiffened panels with openings, only a few references, based on the application of the finite element method, are available, as that of Sivasubramonian et al. [28,29].

Investigation of plates carrying rigidly attached springs or masses has relatively long tradition, from earliest works [30] where the effect of rotary inertia was ignored, over some later Refs. [31,32], where the mentioned rotary inertia effect is taken into account, to newer works simultaneously treating both additional inertia and stiffness attachments [33–36]. However, an assumption on rigid attachment is not always realistic and therefore models for plates carrying elastically mounted lumped mass (one-degree-of-freedom (dof) spring-mass system) have been developed [37–39]. Avalos et al. applied optimized Rayleigh-Ritz method to solve the problem for rectangular and circular plates, [37] and [38], respectively, while Wu and Luo used so called analytical-and-numerical-combined method (ANCM) [39]. Wu offered solution for multiple three-dof systems attached to a plate using equivalent mass method (EMM), i.e. by replacing each three- dof spring–mass system (or substructure) by a set of equivalent masses so that the dynamic characteristics of a rectangular plate (or main structure) carrying any number of elastically mounted lumped masses may be obtained from the plate carrying the same sets of rigidly attached equivalent masses [40]. In [40] advantages of EMM over conventional FEM for the considered problem are also elaborated. To the authors' knowledge, there are no references treating the problem of stiffened panels with spring-mass systems attached.

In this paper an overview of different vibration problems inherent to rectangular plate systems, that can be handled by the energy based assumed mode method [15], is presented. The paper relies on theoretical contributions presented by the authors in recent years on dry vibration of different plate systems [4–9]. Although it is out of this paper scope, it is worthy to mention that above models are successfully coupled with potential flow numerical model to analyse hydroelastic vibration of bottom and vertical plate structures, [41] and [42], respectively. Also, the method was recently extended to treat plates with stepwise thickness [43]. In this paper, as a novelty, the theoretical model is extended to above explained and rather complex problem of vibration of plate structures carrying arbitrary number of spring-mass systems. The receptance method is applied and mathematical formulation is presented in details. Based on the presented theoretical developments an in-house tool for vibration analysis of complex plate systems has been developed and its main features are elaborated. It can handle: thin (Kirchhoff) and thick (Mindlin) plates and stiffened panels, with and without openings, carrying rigidly or elastically connected lumped attachments, respectively, and enables very fast pre- and post-processing. For the forced vibration linear harmonic excitation is assumed meaning that the problem can be easily solved by the mode superposition method if natural response is calculated in advance. A range of illustrative numerical examples is generated, and the results obtained by the developed tool are validated against solutions existing in the relevant literature (where applicable) and general FEM software results [44].

2. Theoretical background

2.1. General

The Mindlin (thick) first-order shear deformation plate theory is adopted in the mathematical model [45,46]. The theory operates with three general displacements, i.e. plate deflection w, and angles of cross-section rotation about the x and y axes, ψ_x and ψ_y, respectively. The governing equations of motions (obtained from the equilibrium of sectional and inertia forces) yield:

$$\frac{\rho h^3}{12}\frac{\partial^2\psi_x}{\partial t^2}-D\left(\frac{\partial^2\psi_x}{\partial x^2}+\frac{1}{2}(1-\nu)\frac{\partial^2\psi_x}{\partial y^2}+\frac{1}{2}(1+\nu)\frac{\partial^2\psi_y}{\partial x\partial y}\right)-kGh\left(\frac{\partial w}{\partial x}-\psi_x\right)=0, \tag{1}$$

$$\frac{\rho h^3}{12}\frac{\partial^2\psi_y}{\partial t^2}-D\left(\frac{\partial^2\psi_y}{\partial y^2}+\frac{1}{2}(1-\nu)\frac{\partial^2\psi_y}{\partial x^2}+\frac{1}{2}(1+\nu)\frac{\partial^2\psi_x}{\partial x\partial y}\right)-kGh\left(\frac{\partial w}{\partial y}-\psi_y\right)=0, \tag{2}$$

$$\frac{\rho}{kG}\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2 w}{\partial y^2}+\frac{\partial\psi_x}{\partial x}+\frac{\partial\psi_y}{\partial y}=0, \tag{3}$$

where ρ represents plate density, h is plate thickness, k is shear coefficient, while ν is Poisson's ratio. Further, D represents plate flexural rigidity $D=Eh^3/(12(1-\nu^2))$, while E and $G=E/(2(1+\nu))$ are Young's modulus and shear modulus, respectively.

The eigenvalue problem of a plate system, Fig. 1, is formulated by Lagrange's equation, requiring calculation of the total system potential and kinetic energies, V and T, respectively. These energies are obtained by subtracting potential and kinetic energies of openings from the corresponding plate energies, while energies of continuous stiffeners or lumped attachments should be added to bare plate energies [4–9]. Therefore, one can formally write:

$$V=V_p+V_s+V_l-V_o, \tag{4}$$

$$T=T_p+T_s+T_l-T_o. \tag{5}$$

where V_p is the plate strain energy, V_s represents the strain energy of