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A localized lagrange multipliers approach for the problem of vehicle-bridge-interaction

Qing Zeng, Charikleia D. Stoura, Elias G. Dimitrakopoulos*

Department of Civil and Environmental Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong

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ABSTRACT

This paper proposes a time-integration analysis scheme for the vehicle-bridge-interaction problem. Key feature is the introduction of artificial auxiliary contact points between the wheels and the bridge deck elements in contact. The artificial points allow the formulation of two sets of kinematic constraints and two sets of contact forces (i.e. localized Lagrange multipliers), between the vehicle and the bridge, that enable the partitioned, non-iterative, dynamic analysis of the two subsystems. To demonstrate the accuracy of the proposed approach, the paper first examines a simple example of a single sprung-mass model traversing a simply supported bridge. Then, it considers a more realistic problem of eight train vehicles crossing an arch bridge. The train vehicles are simulated as multibody assemblies and the bridge with a three-dimensional finite element model. The results prove the computational efficiency of the proposed scheme compared to existing algorithms.

1. Introduction

The role of bridges in contemporary High-Speed Railways (HSRs) is vital. Bridges may constitute even more than 80% of the total length of a HSR line [1]. At operational speeds as high as 300 km/h (or even higher) [1], the dynamic vehicle-bridge-interaction (VBI) becomes critical to the operation safety and the travel comfort of HSRs [2]. From a numerical simulation standpoint, an accurate and computationally efficient analysis of the VBI is essential, especially when the dimensions of the system are large, or a large amount of response-history analyses are necessary.

Most time-integration algorithms developed for the VBI problem, either tackle the coupled vehicle-bridge system (*coupled algorithms*) [2–10], or solve the partitioned vehicle and bridge subsystems separately through iterations (*iterative algorithms*) [11–21]. A common characteristic of the coupled algorithms is that they lead to time-depended system (e.g. stiffness and damping) matrices [2–6]. The time dependency arises from the moving (wheel-rail) contact forces that act as (internal) loads within the global (coupled) vehicle-bridge system. Consequently, the global system matrices must be updated and factorized at each time step, leading to considerable computational effort [20]. In addition, since coupled algorithms solve the equations of motion (EOMs) of the vehicle and the bridge subsystems together, the size of the system matrices is larger than otherwise.

Iterative algorithms avoid the time-dependency of the system matrices. Instead, at each time step they rely on an iterative procedure which terminates when a convergence criterion is satisfied, e.g. a compatibility condition is within a specified tolerance. Xia et al. [11] proposed such a scheme that treats the contact forces as external loads to the vehicle and the bridge subsystems respectively, and solves the two subsystems separately. Liu et al. [12] referred to this algorithm as *loosely coupled iterative algorithm* and applied it to analyse a composite bridge under HSR trains. More recently, Zhang and Xia [13] proposed a refined iterative algorithm, in which the first trial of the vehicle response is predicted by setting the bridge motion to zero.

Neves et al. [20] proposed a different method that also avoids the use of time dependent system matrices. Instead, it solves *directly* a system of linear equations, comprised of the EOMs of the vehicle and the bridge, and the compatibility condition between the two subsystems. The solution returns simultaneously the displacements and the contact forces of the system. That algorithm [20] uses block factorization to exploit specific properties of the differentiation operator matrix, i.e. symmetry, positive definiteness and bandwidth [20].

An alternative approach that potentially incorporates the advantages of both *coupled* [4–6] and *iterative* schemes [11,12] is the localized version of the Lagrange multipliers method (Park et al. [22,23]). This approach originates from the field of computational mechanics and it focuses on coupled systems, e.g. the compressible internal fluid-structure interaction problem [23]. Its novelty is that it introduces an additional artificial auxiliary point at the contact interface, with the aid

* Corresponding author.
E-mail addresses: zengqing@ust.hk (Q. Zeng), cstoura@connect.ust.hk (C.D. Stoura), ilias@ust.hk (E.G. Dimitrakopoulos).
URL: http://ilias.people.ust.hk/ (E.G. Dimitrakopoulos).

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of which, it assigns two sets of Lagrange multipliers (in contrast to the conventional Lagrange multiplier method) and states two sets of kinematic constraints between the auxiliary point and the other contact points. This prevents singularities associated with multi-point constraints and allows an efficient partitioned treatment of the coupled system [22].

Following [22,23], the present study proposes a localized Lagrange multipliers method for the VBI problem. The scheme is both accurate and computationally efficient as it treats the vehicle and the bridge subsystems in a partitioned manner but avoids iterations. It consists of three independent analysis modules: (1) the *interaction solver* that calculates the contact force; (2) the *vehicle solver*, and (3) the *bridge solver* that integrate separately the vehicle and the bridge subsystems, respectively. By means of demonstration, the present study first considers a moving sprung mass model running over a simple bridge, and later a more realistic system consisting of an arch bridge traversed by a series of moving (train) vehicles.

2. Modelling of vehicle subsystem

A typical HSR train consists of a series of vehicles representing the locomotives and the passenger coaches (Fig. 1(a)). Similarly to many recent studies [4–6,19], this paper models each train vehicle as a three-dimensional (3D) multibody assembly. Each vehicle consists of seven rigid body components: one car body, two bogies and four wheelsets. Linear springs and dashpots representing the suspension systems connect the distinct components. This analysis assumes that before the vehicle enters the bridge it travels on a rigid embankment, and that both structures display the same rail irregularities. Thus, the train enters the bridge with non-zero initial deformation and maintains a constant speed. The simulation omits the longitudinal degrees of freedom (DOFs) and considers each vehicle as (dynamically) independent of the other vehicles. The car body and the bogies have five DOFs each, with displacement vector:

$$\mathbf{u}^u = [y^u \quad z^u \quad \psi^u \quad \phi^u \quad \theta^u] \tag{1}$$

where the superscript $()^u$ denotes the car body when $u = c$, while $u = t1, t2$ stands for the front and the rear bogies, respectively (Fig. 1). The DOFs y and z are the lateral and vertical (translational) displacements, and ψ, ϕ and θ are the yawing, rolling and pitching (rotational) Euler angles. Excluding the pitching rotation of the wheelset (Fig. 1(b)), the motion of each wheelset ($u = w$) is described with four DOFs [6]:

$$\mathbf{u}^{wi} = [y^{wi} \quad z^{wi} \quad \psi^{wi} \quad \phi^{wi}] \tag{2}$$

where $i = 1$–4 denotes the number of the wheelset. Thus, the total DOF of each vehicle model is $DOF^V = 31$:

$$\mathbf{u}^V = [\mathbf{u}^c \quad \mathbf{u}^{t1} \quad \mathbf{u}^{t2} \quad \mathbf{u}^{w1} \quad \mathbf{u}^{w2} \quad \mathbf{u}^{w3} \quad \mathbf{u}^{w4}]^\mathrm{T}. \tag{3}$$

The superscript $()^V$ denotes the vehicle subsystem and $()^\mathrm{T}$ indicates the transpose of a matrix throughout the paper. The matrix expression of the EOM for the vehicle subsystem can be written as:

$$\mathbf{K}_\mathrm{eff}^V\mathbf{u}^V - \mathbf{W}^V\boldsymbol{\lambda}^V = \mathbf{F}^V \tag{4}$$

where $\mathbf{K}_\mathrm{eff}^V$ is the effective stiffness of the vehicle:

$$\mathbf{K}_\mathrm{eff}^V = \mathbf{M}^V\frac{d^2}{dt^2} + \mathbf{C}^V\frac{d}{dt} + \mathbf{K}^V. \tag{5}$$

$\mathbf{M}^V$, $\mathbf{K}^V$ and $\mathbf{C}^V$ are the mass, stiffness and damping matrices of the vehicle, the details of which can be found in [19]. $\mathbf{u}^V$ is the displacement vector of the vehicle and $\frac{d}{dt}$ denotes time differentiation (of e.g. the displacement vector). $\mathbf{F}^V$ is the force vector of the vehicle, which consists of the gravity loads. The direction matrix $\mathbf{W}^V$ and the contact force vector $\boldsymbol{\lambda}^V$ are:

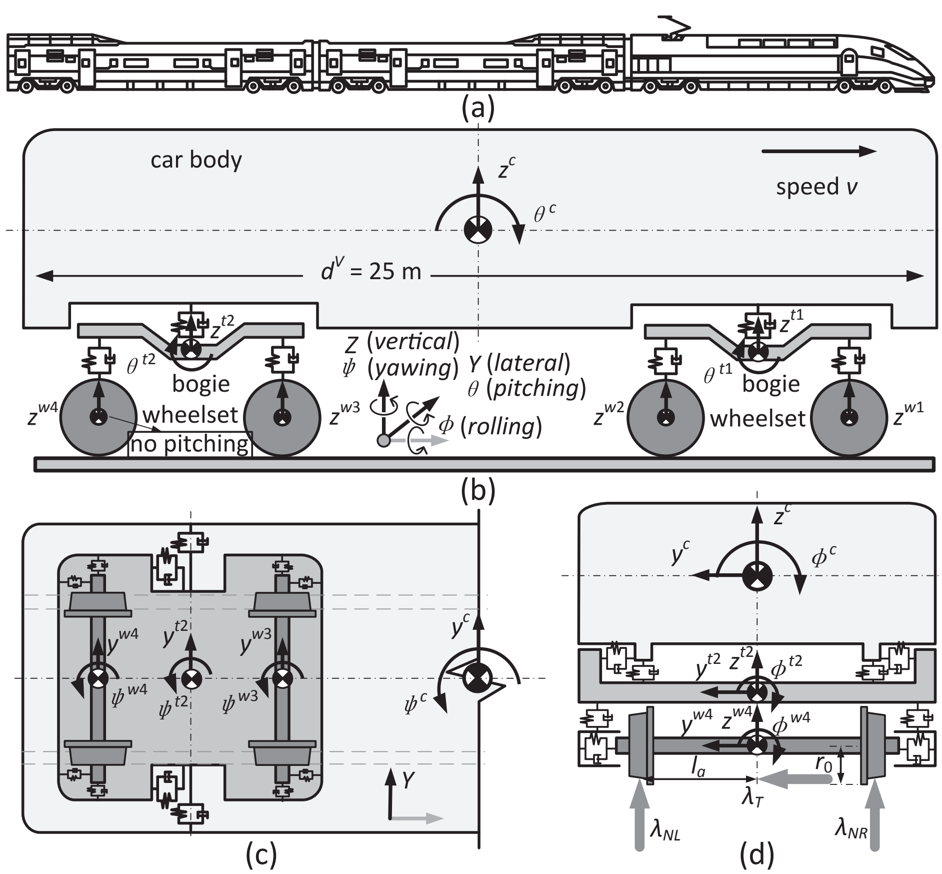

Fig. 1. 3D railway vehicle model: (a) a typical HSR train; (b) side view, (c) top view and (d) back view of the vehicle.