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Pole placement method to control the rocking motion of rigid blocks

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ABSTRACT

In recent years only a very small number of studies analysed the use of active or semi-active control methods in order to avoid the overturning of rigid block-like structures under base excitation. In this paper, an active control algorithm, based on the pole placement method, is developed for rigid blocks starting from a description of the rocking motion through linearised equations. Such an approach is an excellent approximation for slender rigid blocks for which linearised equations can fully describe the rocking motion due to the smallness of the inclination angle. The control method has two objective: the first one is related to the possibility of vanishing the external excitation at each instant, the second one is to make the rest position of the block a stable point. The first analyses revealed the good robustness of the control algorithm to a variation of the sampling time and of the time-delay of the real control devices. Furthermore, the further parametric analyses pointed out the good effectiveness of the proposed control algorithm both in the reduction of the rocking angle and in the protection from the overturning, also in the case of blocks with low slenderness. Overturning spectra are obtained in the case of rigid block with and without active control.

1. Introduction

Rigid block structures like obelisks, statues, big storage boxes and transformers are often subjected to collapse event when they are excited by base acceleration. In the last two decades different techniques have been studied in order to improve the dynamic behaviour of these systems. All these techniques could not have been developed without the fundamental contribution of articles such as [1,2], whose topic is the behaviour of the stand-alone rigid body. These papers have paved the way for further investigation about rigid blocks. Papers [3,4] classified different types of motion (rest, sliding, rocking, sliding-rocking) through behaviour maps, in order to highlight the characteristics of these objects. Other authors focused their attention on the base acceleration used to excite the blocks. Harmonic and one sine excitations [5–7] were chosen because they could represent near-source earthquakes, as it is confirmed by [8,9]. Seismic input are used in [10,11] to obtain valid results only for specific cases.

The majority of the works, usually, consider symmetric rigid blocks while the effects of the eccentricity of the mass of the these objects were examined just in a few articles, such as [12,13].

On the whole, all the previous articles deal with the behaviour of the stand-alone rigid block. On the contrary, in the last two decades, passive methods have been fully examined by different authors. Specifically, in order to protect rigid blocks from overturning, the effectiveness of base anchorages was studied in [14,15], while [16,17] highlighted the efficiency of the base isolated system. Moreover, the effectiveness of the base isolation on the reduction of the dynamic response of an isolated rigid block, placed on a multi-story frame, was investigated in [18]. Instead, Di Egidio and Contento [19] took into account the possibility that the rigid block can perform sliding and rocking motion partially outside the isolated base.

It is worth noting that the introduction of an isolation system is not the sole type of passive control. In fact, Corbi [20] proposed a sloshing water damper in order to control the motion of rigid body. A Tuned Mass Damper (TMD) in the shape of pendulum was used by different authors [21–23]. These papers demonstrated a general improvement of the response of the system with respect to the case without mass damper. In [24–26] the TMD system was modelled as a single degree of freedom oscillating mass running at the top of the block.

In order to improve the dynamic behaviour of a rigid body-like structure, the use of active or semi-active control techniques have been investigated in very few papers. Specifically, the use of semi-active anchorages, as a means to increase the acceleration required to topple a reference block, was studied in [27,28], where a feedback-feedforward or a feedback strategy is used, respectively. The efficiency of a semi-active control is finally studied in [29], in order to improve the seismic behavior of rigid blocks.

In this paper, an active control method, based on a feedback

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strategy and capable of driving a base actuator, is proposed. A control algorithm, based on the pole placement method [30–32], is developed for rigid block by starting from the linearised equations of rocking motion. Such an approach is an excellent approximation for slender rigid block for which the rocking motion can be fully defined by linearised equation due to the smallness of the rocking angle. Since it is assumed that the control device works only when the rocking motion of the body starts, the uplift conditions are the same of a stand-alone block. The control algorithm does not modify the angular momenta of the body, before and after an impact. Therefore, also the impact conditions are the same of the stand-alone body. The proposed control method has two objectives: to vanish the external excitation at each instant and to make the rest position of the block a stable point. The first objective can be realised by providing a control force opposite to the external one to the system. The rest position of the block can be considered a stable point if the Jacobian matrix is modified in such a way that the eigenvalues have negative real part.

The paper is organized in six sections. Specifically, after the Introduction, the equations of motion, the uplift and the impact conditions of the controlled block are derived; after that, the control law based on the Pole Placement Method, the evaluating of the optimal control coefficient and of the robustness of this algorithm is performed; the effectiveness of the proposed control method is investigated in the reduction of the rocking amplitude and in the ability to avoid the overturning; finally, an extensive parametric analysis is performed in order to evaluate the efficiency of the control method in blocks with different geometrical characteristics. In the Conclusions, a brief summarizing of what is done in the paper and of the main results obtained is reported.

2. Controlled mechanical system

The mechanical system considered in this paper is constituted by a symmetric parallelepiped, whose geometrical parameters are the heigh H, the base B and the distance R between one of the base corners and the mass centre G. Moreover the other base dimension is equal to the unity. The mass of the block is $M = \rho \times B \times H \times 1.0$, where the mass density ρ is $\rho = 1800\ \mathrm{kg/m^3}$ in the following. The body is posed on a support that translates horizontally and has a negligible mass with respect to the one of the block. It is connected to the ground by an actuator, that provides the control force $u(t)$. Quantity $\ddot{x}_g(t)$ is the external base acceleration (Fig. 1a). Due to the sufficiently large slenderness $\lambda = H/B$ ($\lambda \geqslant 3$), no sliding motion is allowed. So the block can undergo only rocking motion. As a consequence, the sole Lagrangian parameter necessary to describe the motion, is the angle $\vartheta(t)$, which measures the rocking amplitude. Fig. 1b and c show when the angle $\vartheta(t)$ is positive or negative. There are only two possible phases of motion: the full-contact phase and the rocking one.

The angle α_{cr} is the value of $\vartheta(t)$ in which the block reaches its unstable statical position. In fact, in this case, the vertical projection of the mass centre G passes through the rocking corner.

2.1. Governing equations of the controlled system

The rocking equations of a rigid block, subjected by base excitation, are well-known in Literature [11,8]. The system showed in Fig. 1 can be described by the same equations of a stand-alone body with the additional term of the control force represented by the term $MR\cos(\alpha_{cr} + \vartheta(t))\frac{u(t)}{M}$. The control force $u(t)$ applies an acceleration $u(t)/M$ at the base of the block. Since this acceleration works as the external one, it appears in the equations of motion as a contribution added to external acceleration $\ddot{x}_g$. They read:

$$\begin{aligned}
&(J_G + MR^2)\ddot{\vartheta}(t) + gMR\sin(\alpha_{cr}-\vartheta(t))-\\
&MR\cos(\alpha_{cr}-\vartheta(t))\left(\ddot{x}_g(t) + \frac{u(t)}{M}\right) = 0\\
&(J_G + MR^2)\ddot{\vartheta}(t)-gMR\sin(\alpha_{cr} + \vartheta(t))-\\
&MR\cos(\alpha_{cr} + \vartheta(t))\left(\ddot{x}_g(t) + \frac{u(t)}{M}\right) = 0
\end{aligned} \tag{1}$$

where g is the gravity acceleration and J_G is the polar momentum evaluated with respect to the mass centre G. The equations represent the dynamical equilibrium of the moments around the left base corner (first equation of Eq. (1)) and around the right base corner (second equation of Eq. (1)).

The control algorithm is obtained by linearising the equations of rocking motion with respect to the variables $\vartheta(t)$. This assumption leads to an excellent approximation in the case where the rocking angle $\vartheta(t)$ is very small. In fact, the rocking angle of very tall block has to be smaller than the very little critical angle α_{cr}, in order to make the motion stable and to avoid the overturning. By expanding in Mc Laurin series up to the first term $\sin(\alpha_{cr} \pm \vartheta(t))$ and $\cos(\alpha_{cr} \pm \vartheta(t))$ and letting $\sin(\vartheta(t)) \approx \vartheta(t)$ and $\cos(\vartheta(t)) \approx 1$, the linearised rocking equations in

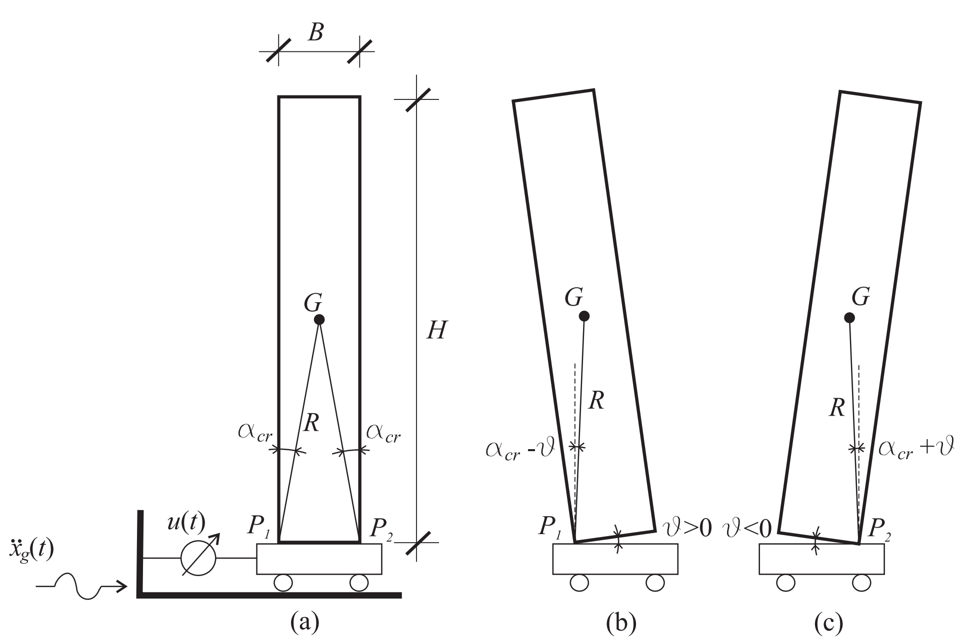

Fig. 1. Rigid block: (a) Geometrical characterization; (b) Rocking around the left corner P_1; (c) Rocking around the right corner P_2.