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A robust frame element with cyclic plasticity and local joint effects

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ABSTRACT

A robust elasto-plastic element is developed for analysis of frame structures. The element consists of a beam member with end joints with properties permitting representation of the effect of section forces in adjoining members, like axial forces. By use of the equilibrium formulation the deformations of beam member, plastic hinges and joints become additive and can be expressed in explicit form. The plastic deformations of the beam and the joints are represented by separate plastic mechanisms, described by the same generic cyclic plasticity format. This format is defined by an energy function, a yield surface, and a plastic flow potential for each plastic mechanism. In the cyclic plasticity model each component is characterized by the elastic stiffness, the yield capacity, the additional flexibility at initial yield, the ultimate capacity and a shape parameter describing the curvature of the hysteresis curve. The yield surface is represented by a recently developed generic format, combining the section forces into a homogeneous function of degree one and permitting smooth transition between regions with large and more moderate curvature. A robust return algorithm of approximately second order is developed, using a mid-step state to obtain representative information about the return path. The element is implemented in a co-rotational large-deformation computer program for frame structures. The formulation is illustrated by application to a couple of typical offshore frame structures, and comparison of different representations of the plastic effects illustrates the importance of a robust element with realistic representation of the cyclic plastic mechanisms.

1. Introduction

In the design and analysis of frame structures, e.g. offshore tubular structures and steel frame buildings exposed to earthquakes, a large number of load cases are analysed to ensure the structure can withstand the external loading. Some important load cases involve substantial deformation of members in the elasto-plastic regime, followed by subsequent unloading introducing a need for an accurate representation of the cyclic plastic behaviour of beam members. The cyclic elasto-plastic response of a single beam member has been experimentally investigated in e.g. [1–5] for both uni-axial tension/compression and uni-axial bending, and elasto-plastic cyclic column-buckling of tubular steel columns was investigated and characterised by [6]. While cyclic plastic bending is dominated by the non-linearity in the material behaviour, cyclic column-buckling is characterised by the non-linearity in the material behaviour and in the geometry. Common to both cases is that the plastic deformation is local in the form of plastic hinges, suggesting that it is possible to separate geometric and material non-linearity by proper modelling. The localized plastic deformation in the form of plastic hinges is also observed in full structures [7–9], where cyclic loading of the local members comes naturally via global unloading or load-shedding caused by buckling or plasticity in other members. In full structures the plastic hinges may be caused by a plastic mechanism in the local member itself or by a plastic mechanism at the local joint connecting the structural member to the rest of the structure. It is necessary to distinguish the two types of mechanisms from each other and to acknowledge that they may both be present at the same time at the same location. Experimental investigation of capacities of local joints in tubular structures has been carried out in [10] and extensively characterized in [11].

In addition to plastic mechanisms, in practice the local joints between members introduce additional flexibility in the structure compared to completely rigid connections. The difference in the response of a structure modelled with and without local joint flexibility is clear in both traditional analysis of frames [12] and in bifurcation and stability analysis of frames [13]. Multiple experimental programs have investigated and characterized local joint flexibility [14–16], essentially describing the additional flexibility of the local joint by parametric equations depending on the local joint geometry. Recently detailed finite element models have been used to develop such parametric

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equations after validation with experimental data [17–19]. The inclusion of local joint flexibility in analyses has primarily been modelled by separate elastic joint elements [20–22], introducing a need for a transformation between flexibility and stiffness and most often introducing infinite stiffness terms for displacement directions with zero joint flexibility. Separate joint elements including both elastic and plastic flexibility have been introduced [23], introducing this infinite stiffness problem. In some finite element codes e.g. RONJA developed by Rambøll, the local joint flexibility has been introduced in the member elements by static condensation, a method that does not resolve the problem with infinite stiffness.

In most frame structures the imperfections of the individual members need to be modelled to account for the effect of the normal force. Imperfection effects were introduced by [24,25] in an element with negligible shear flexibility based on parabolic and a sine imperfection shape respectively. An explicit elastic element including shear flexibility and a parabolic imperfection shape was introduced in [26] and was extended to include plastic mechanisms in the form of concentrated plastic hinges at the ends of the elastic beam giving an explicit elasto-plastic beam element with initial imperfections.

The differences between beam elements with concentrated plastic hinges and beams modelled with spread of plasticity using fibre elements was investigated in [27], finding the relative magnitude of the generalized plastic strain components to be similar for the two types of models. Several element formulations with concentrated plastic hinges have been proposed, some having three possible plastic hinges [28–30] with one hinge located at mid-span to account for column buckling effects. The degrees of freedom associated with the mid-point plastic hinge are typically removed by static condensation. Other elements, primarily used for column problems, have been suggested [31] with only a hinge at mid-span, and a proposal for softening hinges with location dependent on the section force distribution in [32]. The difference between displacement, flexibility and mixed formulations of beams was investigated in [33] finding the flexibility format quite accurate taking into account its low-order modelling compared to higher-order modelling typically used in displacement and mixed formulations. Flexibility formulations via a 6×6 equilibrium format was proposed in [34,35] for monotonic and cyclic plasticity models as well as in [26] including local imperfections. In order to model cyclic plasticity in frame structures more accurately [36] introduced a generalized formulation of the cyclic plasticity model from [37]. The model is based on non-linear kinematic hardening rules and evolution of the model parameters and was subsequently extended to include local joint plastic mechanisms [38]. Common to all of the element formulations is that they are based on a set of yield functions bounding the elastic domain and a set of plastic flow potentials to describe the development of plastic deformation.

The yield surface of the individual plastic mechanisms may be determined either by approximate analytical methods [39] or numerical estimates [40] and subsequently modelled in various ways. A standard approach that ensures convexity of the yield surface is the use of multi-linear yield surfaces. However, the checks of multiple surfaces and determination of gradients at vertices may be difficult, see e.g [41]. To overcome the difficulty with multiple checks, single-equation formulations of yield surfaces have been proposed, e.g. higher-order polynomial approximations [42,43], NURBS-based formulations [44] or use of Fourier principles [45]. All these have the disadvantage that the coefficients in the equations or locations of the control points may be difficult to determine while simultaneously ensuring convexity of the yield surface. The convexity was ensured in a surface format proposed by [46,47] using a Minkowski sum of ellipsoids, and the use of the convexity of the ellipsoids was utilized by [36] to form a generic convex single-equation yield function without the need to form the actual Minkowski sum. For some cyclic plastic deformation histories the shape of the yield surface has been found to change, and a weighted average of different yield surfaces has been applied with success [48,49].

Independent of the choice of the yield surface formulation it is desirable to be able to make large load/deformation increments in order to have efficient computations. The analysis procedure typically determines the displacement increments via a global analysis and subsequently determining the element deformations and forces, ensuring that the yield condition is not violated in the individual elements. Satisfaction of the yield condition is typically attained by a return algorithm where combinations of the deformation evolution equations and the yield conditions determine the correct increment in element forces. For continuum elements [50] proposed a return algorithm for plane stress elasto-plasticity including the algorithmic tangent stiffness needed to ensure second order convergence of the global solution. While the plane-stress elasto-plasticity return algorithm was developed for a fairly simple yield surface, a more advanced algorithm was developed for structural concrete with a more complicated yield surface [51] making use of sub-stepping techniques as well as line search to ensure a proper return to the yield surface. In geotechnics the yield surface is typically divided into multiple domains and several return algorithms have been developed to overcome the problems with finding the correct domain to return to [52–54]. Where [52] modified the individual domains, [53] used bisection in a transformed space and a combination of returning to an unhardened state and subsequently returning to the hardened state, and [54] made use of a relaxation technique to obtain a more robust algorithm. The efficiency of the return algorithm may in some cases be increased by transforming to an invariant space [55] combined with multi-linear yield surfaces and defining separate rules for return to vertices [56]. Separate algorithms have also been developed for coupled problems including damage [57]. Common to all return algorithms is that they need to be quite robust to allow for large increments of deformation in any direction, and for plastic hinges it is paramount to ensure the robustness of the algorithm independently of the given yield surface.

This paper develops an elasto-plastic frame element, and introduces plastic beam hinges and elasto-plastic joints via the concept of additive flexibilities. The element is defined in an equilibrium-based co-rotational formulation and is sufficiently general to encompass elastic element formulations ranging from standard cubic shape functions to normal force dependent stiffness functions including initial member imperfections, see e.g. [26,58], as well as plastic mechanisms ranging from ideal plasticity to models coupling elasto-plasticity and damage. The cyclic plasticity formulation proposed in [37] is generalized and extended to ensure invariance for doubly-symmetric beam cross sections. The yield function is of the type proposed by [36] and determination of parameters as well as gradual change of shape and inclusion of shear effects are discussed. A novel two-step return algorithm that includes the effects of distributed loads is introduced and shown to increase the robustness of traditional single-step return algorithms considerably. Finally, examples of realistic tubular offshore structures are used to illustrate the effect of the plasticity formulation as well as the robustness of the equilibrium element formulation and the modified return algorithm. The examples highlight the differences between standard element and plasticity formulations and the present integrated formulation, illustrating the necessity of having an accurate representation of cyclic plasticity and local joint mechanisms.

2. Elasto-plastic frame element

The frame element is defined in an equilibrium format with six deformation modes with energy conjugate section forces as illustrated in Fig. 1. Details of the equilibrium formulation may be found in [26,58]. The deformations and the section forces are arranged in the vectors

$$\tilde{\mathbf{u}}_t = [u,\varphi_x,\varphi_{z1},\varphi_{z2},\varphi_{y1},\varphi_{y2}]_t^T, \tag{1}$$

$$\tilde{\mathbf{q}}_e = [\widetilde{N},\widetilde{T},\widetilde{M}_{z1},\widetilde{M}_{z2},\widetilde{M}_{y1},\widetilde{M}_{y2}]^T, \tag{2}$$