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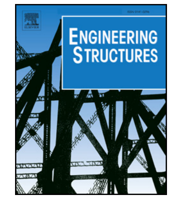

A novel cable element for nonlinear thermo-elastic analysis

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ABSTRACT

The exact solution of inextensible catenaries in Cartesian coordinates is utilized to propose an efficient two-node cable element for static analysis of three-dimensional cable structures. This element can consider out of plane inclination without using any transformation matrices. Since the element is formulated within the framework of large curvature assumption, cables with large sag, as encountered in long-span cable-stayed bridges and suspension bridges, can be modeled accurately. The proposed element also accounts for the thermal effects. By defining the stiffness component as the ratio of infinitesimal load increment to infinitesimal increase in length, explicit entries of the tangent stiffness matrix are derived through equating the total differentiation of the strained length and the elastic elongation of the cable. The tangent stiffness matrix is available in a closed form and the need of taking the inverse of the flexibility matrix, which is faced in the solution procedure of elastic catenary, is eliminated. The robustness of the suggested technique is established through investigation of significant case studies, including slack and pre-tensioned spatial cable networks. Excellent agreement between the present results and those found in the literature indicates the versatility of the proposed scheme.

1. Introduction

Over the past two centuries, analysis and design of cable-supported structures have received huge attention as a crucial topic in the mainstream of scientific research. Owing to their unique mechanical and aesthetic features, cables are widely applied as constituent parts of many engineering structures, such as, suspension roofs, long-span suspension bridges, cable supported bridges and power transmission lines. Cables are flexible members that exhibit highly nonlinear behavior when subjected to external loads. This structure, within a cable-supported body, undergoes large displacements and rotations and sustains significant portions of load. Pretension is proposed as a simple technique to alleviate the deflection of cable structures. Numerous studies can be found in the literature addressing various schemes for investigation of the behavior of cable structures. In fact, the cable members have been widely modeled, based on two different approaches, namely the finite element method with interpolation functions and also the analytical approach which makes use of explicit expressions of a catenary.

In the first scheme, a cable is represented by a number of two-node, multi-node or generally curved elements. The displacement field within the element domain is approximated using the interpolation functions. In 1965, Ernst suggested that a cable member can be modeled by truss elements for the first time. He also introduced a modified axial stiffness to account for the sag effects of a hanging cable [1]. Although his method provided satisfactory results in some cases, it was rather inefficient since a large number of truss elements was required to achieve an acceptable level of accuracy. Later, Knudson embarked on the improvement of this method in 1971 [2]. Various researchers have further developed the truss element by introducing the nonlinear behavior and various loading conditions [3,4]. Besides, different types of two-node elements with rotational degrees of freedom have been proposed by several researchers [5–7]. The cable members have been also modeled based on the isogeometric approach with Lagrangian shape functions. In this method, the shape of an infinitesimal cable element is approximated using multi-node curved elements [8,9]. Wu and Su implemented a Four-node isogeometric element for analysis of cable structures [10]. In 2013, a six-node isogeometric element was proposed by Wang et al. [11]. The main drawback of the isogeometric elements in modeling of cable assemblies is their complexity and large number of degrees of freedom. This makes the analysis laborious and significantly time consuming. Further, since the explicit form of the tangent stiffness matrix is not available, numerical approaches must be iteratively adopted to derive the tangent stiffness matrix. In some cases, such analysis approaches lead to the numerical instabilities [12].

On the other hand, an element based on the analytical expressions of the elastic catenary was first utilized by O'brien and Francis [13]. They showed that each cable member within a cable structure can be modeled using a single analytical element. In this method, the overall equilibrium of a stretched cable element is satisfied in the Lagrangian

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coordinates, and the exact profile is derived by imposing the boundary conditions at the end of the cable. Many researchers have developed the elastic catenary element by introducing thermal effects and different loading types [14–20]. Salehi Ahmad Abad et al. proposed an extended three-dimensional catenary element which takes the thermal effects and distributed lateral loads in different directions into account [21]. Naghavi Riabi and Shooshtari implemented the elastic catenary along with the Ramberg-Osgood stress-strain relationship to investigate the effects of material nonlinearity on the behavior of cable networks [22]. Recently, Crusells-Girona et al. have employed a mixed variational approach in curvilinear coordinates based on the elastic catenary expressions to model cables with material and geometric nonlinearity [23]. Moreover, a number of researchers have adopted the parabola approach for analysis and design of practical cable structures. Since the parabola approach disregards the large sag effects, it provides an approximate solution to the hanging cables. It is proved that the error of the method increases by increasing the sag to span ratio. Therefore, this approach is unsuitable for modeling deep cables [24–26].

In addition to the aforementioned finite element approaches, many researchers have developed innovative ways for nonlinear analysis of cable structures. Lewis employed the principle of minimum total potential energy along with the dynamic relaxation method to assess the efficiency of pure numerical approaches in analysis of pre-tensioned cable nets [27]. A two-link structure was utilized by Kwan to develop a simple technique for nonlinear analysis of pre-tensioned cable structures. In this approach, similar to a spatial truss, the nonlinear equilibrium equations were written for each node, and then, they were solved by using an iterative method [28]. Stefanou and Moossavi Nejad minimized the total potential energy of the entire structural assembly by the conjugate gradient method to obtain the equilibrium state of the cable structures [29]. The efficiency of various dynamic relaxation methods in analysis of cable structures was studied by Hüttner et al. [30]. To model single-span cables considering extensibility and thermal strains, the finite difference approach was applied by Bouaanani et al. [31,32].

Although the elastic catenary provides highly accurate results, the tangent stiffness matrix is not explicitly available. Therefore, a complicated iterative procedure must be adopted to determine the nodal forces and establish the flexibility matrix. To perform a very systematic analysis, the inverse of the flexibility matrix must be also computed to obtain the stiffness matrix. Thus, many difficulties arise during the analysis procedure of the elastic catenary. On the other hand, simplified cable approaches, such as elastic parabola or elastic straight shape, are problematic in addressing the large sag effects in the deep cables. Moreover, these elements must be transformed from a local axis to the global one via transformation matrices to be able to consider inclination. This action further increases the computational complexities. To improve these drawbacks, a three-dimensional cable element is proposed in this study for static analysis of the general cable structures. The elemental shape considers inclination without using transformation, and it takes both the large sag and thermal strain effects into account. To make the nonlinear analysis easier, the components of the tangent stiffness matrix are presented by relatively simple closed-form expressions. Since the profile of the hanging cable is given by hyperbolic functions, for convenience, the proposed element is referred to as the 'elastic hyperbola'. The numerical outcomes of the studied problems reveal the accuracy and efficiency of the present element in the nonlinear analysis of spatial cable structures.

2. Formulation of the hyperbolic cable

The configuration of a perfectly flexible and elastic cable element stretched between two nodes, namely i and j, is depicted in Fig. 1. As it can be seen, the projected lengths along the x, y and z directions are designated l_x, l_y and l_z, respectively. Further, the nodal forces and nodal displacements along the global axes, initial unstrained length and the self-weight per unit unstained length are denoted by F, u, S and w, in a respective manner.

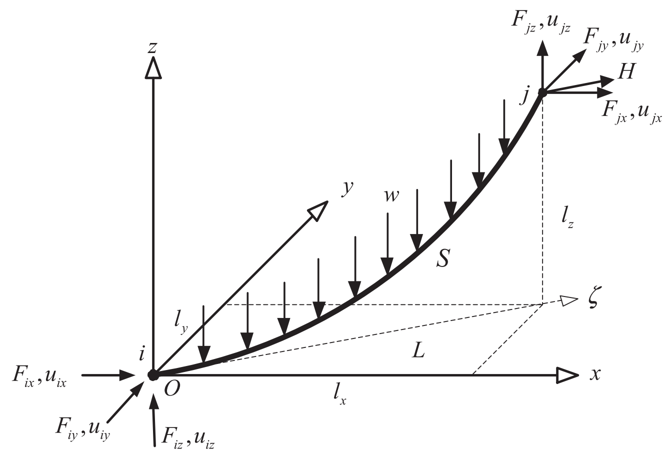

Fig. 1. Configuration of the hyperbolic cable element under self-weight.

The cross-sectional area, elastic modulus and thermal expansion coefficient of the cable are supposed to be constant, and the formulation is developed within the framework of small strains. Note that L and H correspond to the horizontal projected length and the horizontal force of the cable along the local axis, ζ, respectively. Herein, it is assumed that the profile of the cable is sufficiently deep. In other words, no limitations are imposed on the curvature of this structural element. It is worth mentioning that removal of this assumption leads to the simplified elastic parabola approach. Based on the foregoing hypotheses, the profile of the cable hanging under its self-weight with respect to the ζ axis can be defined by the following hyperbolic function [17]:

$$z(\zeta) = \frac{H}{w}\left[\cosh\left(\frac{w}{H}\zeta + \xi\right) - \cosh(\xi)\right] \tag{1}$$

It should be added that satisfaction of the boundary conditions, i.e. $z(0) = 0$ and $z(L) = l_z$, will lead to the subsequent constants:

$$\xi = \sinh^{-1}\left(\frac{\lambda l_z}{L\sinh(\lambda)}\right) - \lambda \qquad \lambda = \frac{wL}{2H} \tag{2}$$

Another parameter is defined as $L = \sqrt{l_x^2 + l_y^2}$. It can be easily shown that the cable tension at the Cartesian coordinates has the next expression:

$$T(\zeta) = H\sqrt{1 + \left(\frac{dz}{d\zeta}\right)^2} = H\cosh\left(\frac{w}{H}\zeta + \xi\right) \tag{3}$$

where T is the tension of the cable. After deformation, the strained length of the cable element can be obtained as:

$$P = \int_0^L \sqrt{1 + \left(\frac{dz}{d\zeta}\right)^2}\, d\zeta = \sqrt{\left(\frac{L}{\lambda}\sinh(\lambda)\right)^2 + l_z^2} \tag{4}$$

where P stands for the strained length of the cable element. Since a real cable has finite axial flexibility, the inextensibility condition must be relaxed to obtain its elastic elongation. For an extensible cable with constant material properties, the Hooke's law is held:

$$\varepsilon(\zeta) = \frac{T(\zeta)}{EA} + \alpha\Delta\vartheta \tag{5}$$

where ε, E, A, α and $\Delta\vartheta$ refer to the cable strain, elastic modulus, cross-sectional area, thermal expansion coefficient and uniform variation in the temperature, respectively. Substituting for the cable tension from Eq. (3) into Eq. (5) and performing some mathematical manipulations yield: